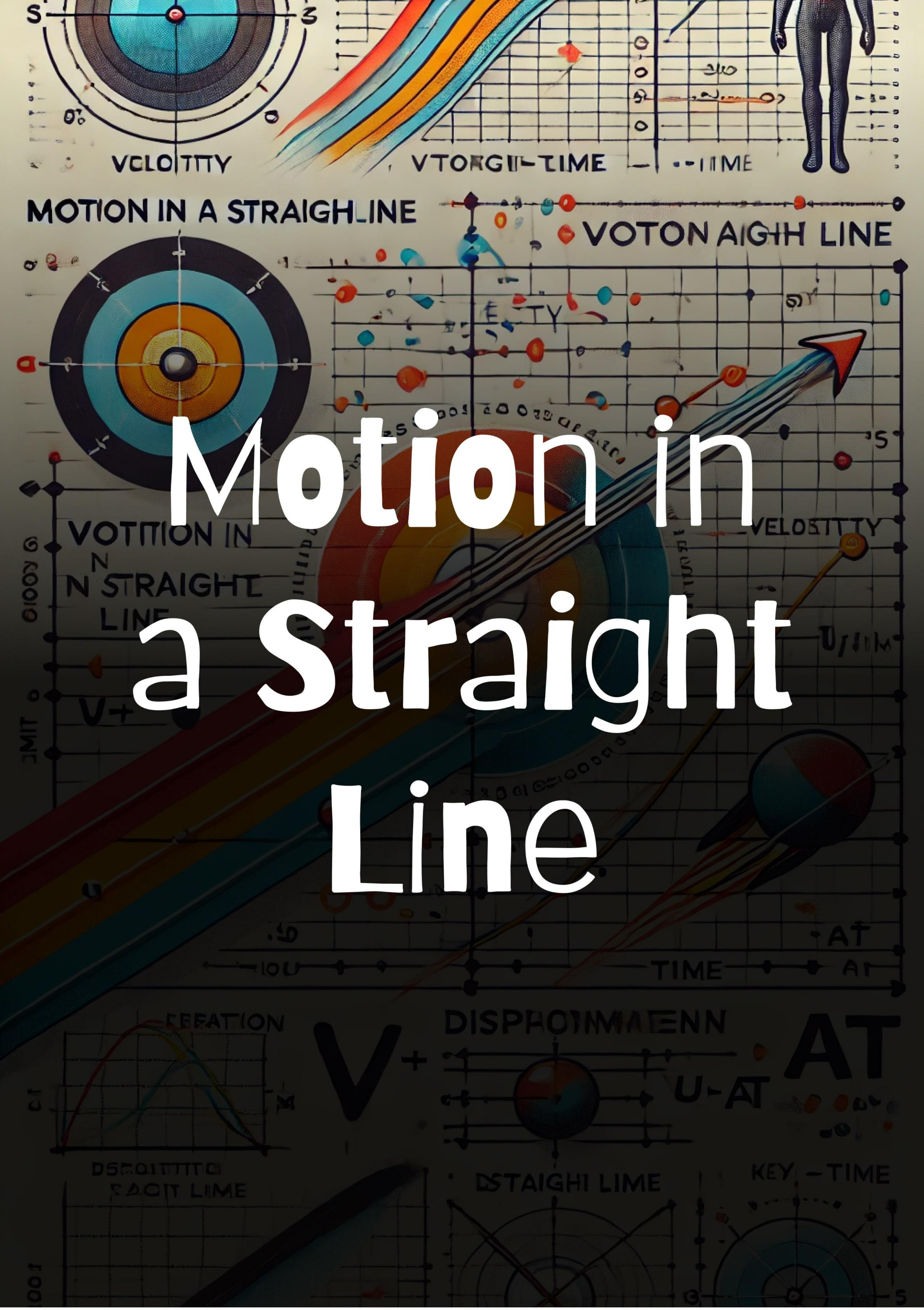
Motion in a Straight Line



Kinematics

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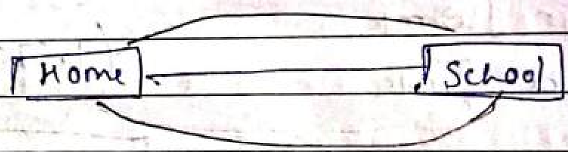
Distance	Displacement
How far (magnitude)	(How far / direction)
→ Scalar (no direc)	→ Vector
→ Actual path travelled	→ change in Position ($x_f - x_i$)
→ total path length	→ Shift in Position
→ dist diff in all three path always true	→ Straight line from initial to final
→ distⁿ depend on path taken	→ displacement is same in all three part
→ We always add distance	→ disp can decrease
→ Can't decrease with time	→ dispⁿ can be +ve, -ve / direct
→ always Positive can't neg	
Unit - metre	- Vector, State function



Path length - (Distance)

Total distance travelled by an object from initial to final position

Displacement :→ Shortest distance b/w initial and final position.

Which of the following option is correct for motion in 1-D

distance = |displacement|

distance > |displacement|

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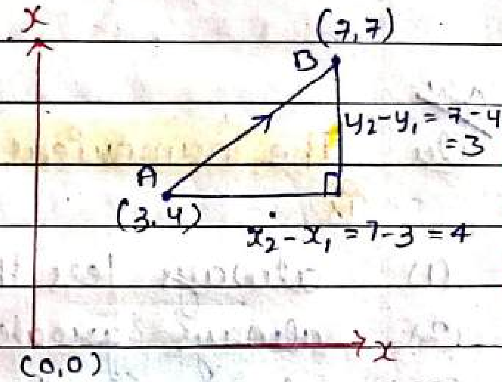
distance | displacement | - object is moving in 1-D without change In dirn then → |dispn| = dist

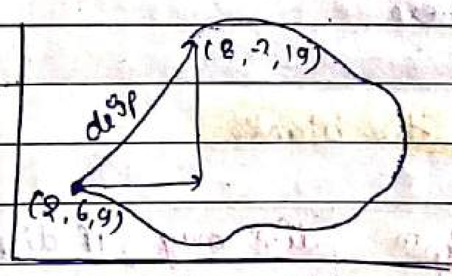
distance ≤ displacement - which of the following option is always wrong

Q total distn b/w 'A' & 'B'



Ans. Can't be Calculate because we don't now actual Path
but Can Calculate dispn because we know final inital Position.

Que If initial position of object (2, 6, 9) and final position (8, -2, 19) then find displacement and distance.



$disp = (8-2)\hat{i} + (-2-6)\hat{j} + (19-9)\hat{k}$

$disp^n = 6\hat{i} - 8\hat{j} - 10\hat{k}$

$|disp| = \sqrt{(6)^2+(8)^2+(10)^2}$

$\sqrt{200} = 10\sqrt{2}$

Qu The magnitude of displacement may or may not be equal to the path length travelled by and Object. in One de two de

$|disp| = dist^n$ = when object is not changing its direction.

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Qu The magnitude of the displacement for a Complete motion may be zero but the Corresponding path length is not zero

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Qu The numerical ratio of displacement to distance is.

(1) always less than 1

(2) always greater than 1

(3) always equal to 1

(4) May be less than 1 or equal 1

$$\frac{|disp^n|}{dist^n} \le 1$$

$$\boxed{\frac{dist}{|disp^m|} \ge 1}$$

Qu Correct Statement among the following.

(1) When $disp^m$ is zero, distance travelled is not zero

(2) When disp is zero, distance travelled is also zero

(3) When distance is zero, displace is not zero

(4) Distance travelled and disp. are always equal.

(5) None of these

Imp

Fill in the blanks

Displacement	$disp^m$ must be zero	$disp^m$ may or may be non zero	If $disp^n$ is zero, then	If disp is not equal to zero, then
Distance	If distance is zero, then?	If $dist^n$ is not equal to zero, then	dist may or may not be zero	disp must be not equal to zero

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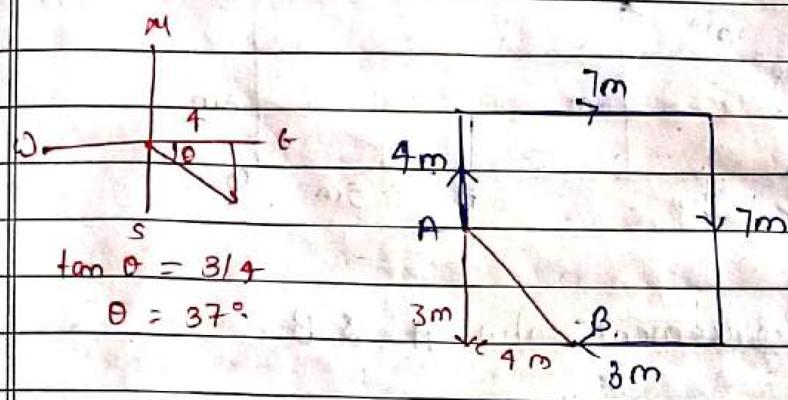
Qu. Find displacement b/w A & B



$\tan\theta = 3/4$

$\theta = 37°$

$$disp = \sqrt{(3)^2+(4)^2}$$
$$= 5$$

disp = 5m (East से 37° South की ओर)

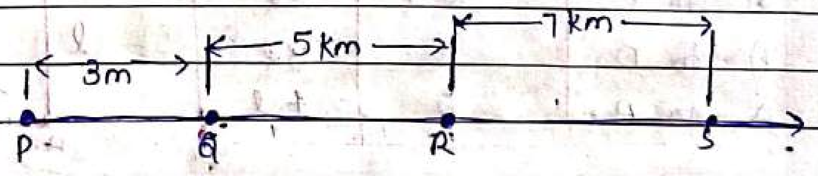
$= \boxed{5m,\ 37° \text{ South of East}}$

II dist = 21 m

Qu. A car moving along in a straight highway from point P to point Q to point R and to point S, then back to point Q and finally to the point R as shown in the figure back.

(a) Find the distance travelled by car

(b) Find the displacement of the car.



(a) dist = 15 + 12 + 5
= 32 m

(b) disp = 8 m

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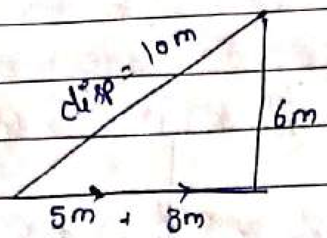
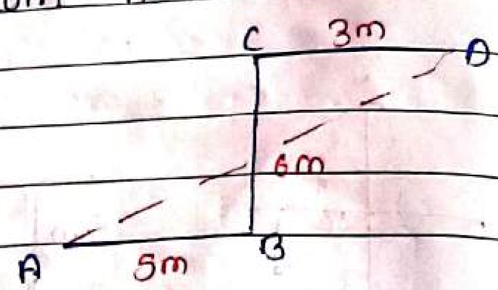
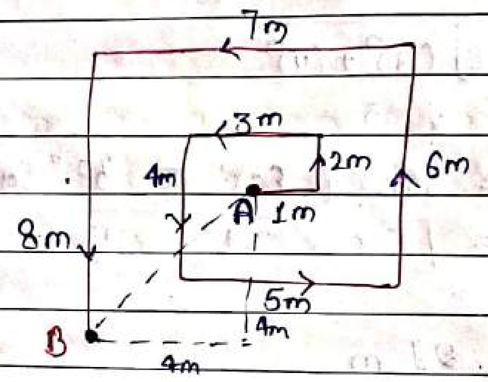
Find displacement from A to D.





Que Find displacement b/w A & B.

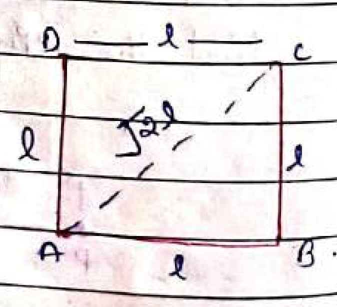


disp = $4\sqrt{2}$

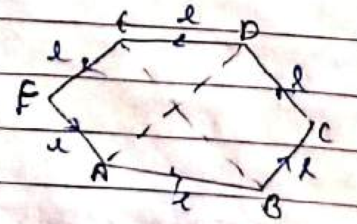
dist = 36m

Que Find displacement and distance

Motion	Displace	Distance
A to B	l	l
A to C	$\sqrt{2}\,l$	$2l$
A to D	l	$3l$
A to A	0	$4l$



Que



Motion	disp	dist
A → B	l	l
A → C	$\sqrt{3}\,l$	$2l$
A → D	$2l$	$3l$
A → E	$\sqrt{3}\,l$	$4l$
A → F	l	$5l$

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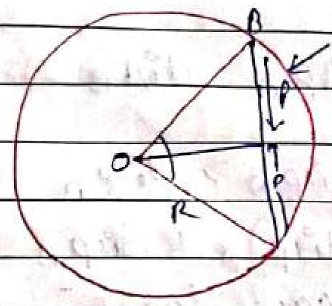
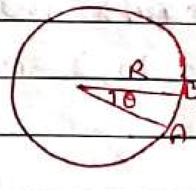
Distance and displacement On Circular Path



$\sin\left(\frac{\theta}{2}\right) = \frac{P}{H} = \frac{P}{R}$

$P = R\sin\left(\frac{\theta}{2}\right)$

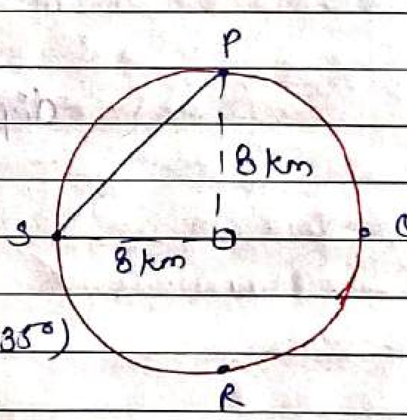
$\boxed{\text{disp} = 2P = 2R\sin\left(\frac{\theta}{2}\right)}$



← Arc = dist = Rθ

← disp = Rθ

A Vehicle moves from Point P to Q to R to S in a Circular Path as shown in the below figure. find the dist travelled by the vehicle
find out the magnitude of the displacement of the Vehicle.



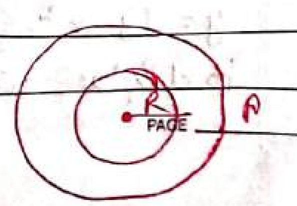
$\text{dist} = R\theta = \frac{3\pi R}{2}$

$\text{disp} = 2R\sin(\theta/2)$

$2R\sin\left(\frac{270}{2}\right) = 2R\sin(135^\circ)$

$2R\frac{1}{\sqrt{2}} = \sqrt{2}R = \sqrt{2}\times 8\ \text{m}$

An Athlete Cover 3 rounds on a Circular track of radius 50 m. Calculate the total dist and disp travelled by him



disp = 0
dist = $6\pi R$ m

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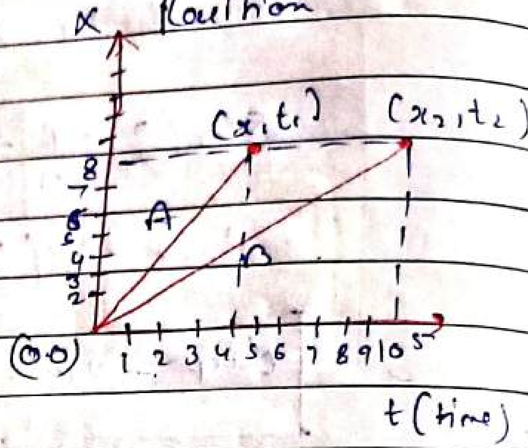
Two man are moving along x-axis as shown in figure the compare

(a) $disp_A = disp_B$

(b) $disp_A > disp_B$

(c) $(disp)_A < disp_B$

(d) cant say



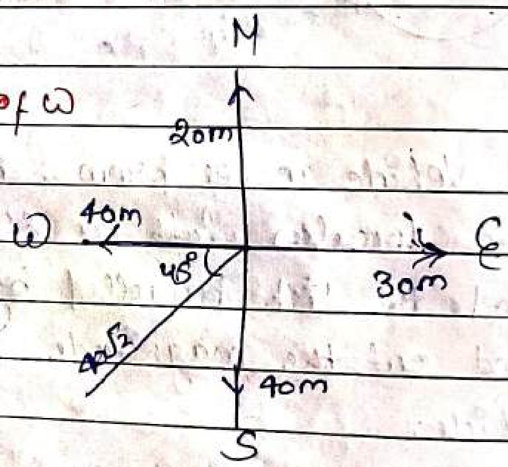
Qu A Person moves 20m towards north then 30 m towards east and finally $40\sqrt{2}$ m south-w His displacement

(a) $10\sqrt{5}$ m $\tan^{-1}(2)$ S of W

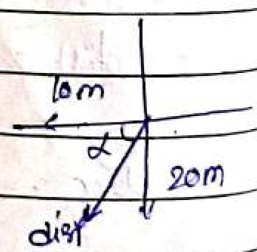
(b) $20\sqrt{5}$, $\tan^{-1}$

(c) 20 m , S-W

(d) $10\sqrt{5}$ m , S-W



$$disp = \sqrt{(10)^2 + (20)^2}$$
$$\sqrt{500}$$
$$10\sqrt{5}$$



$$\tan\alpha = \frac{P}{B} = \frac{20}{10}$$

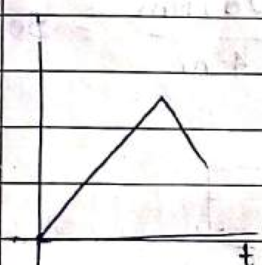
$$\boxed{\alpha = \tan^{-1}(2)}$$

South of West

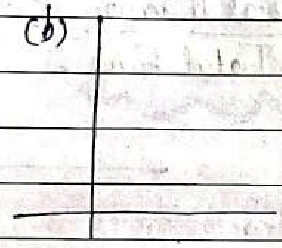
Qu Ram is moving on a path given by equation $y = \sqrt{9 - x^2}$, What would be the ratio of his distance to displac when he travel from $x = -3$ to $x = +3$ m

Qu. Which of the following graph is correct for $dist^n$

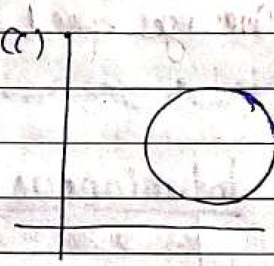
(a)



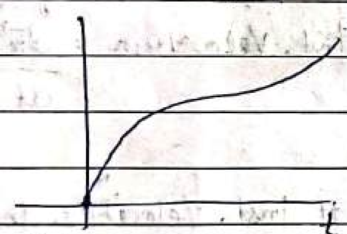
(b)



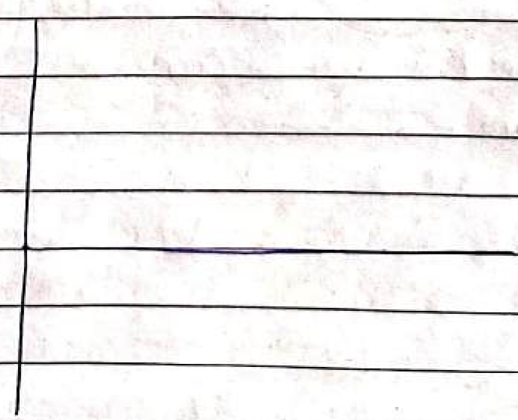
(c)



(d)

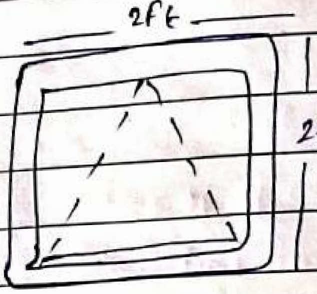


Qu. The position time graph for an elevator travels up and and down is given below. Find the distance and displacement of the elevator b/w 6 second and 21 second.



On a Carom-board (2ft x 2ft). A Striker is shot from one Corner and after striking the front wall It goes into the hole on another Corner. the displacement and distance of Striker is.

(a) 2ft, 2ft ✓
(b) 2ft, $2\sqrt{5}$ ft
(c) $2\sqrt{5}$ ft, 2ft
(d) $2\sqrt{5}$ ft, $2\sqrt{5}$ ft



for Uniform

Average Speed = $\frac{\text{total distance}}{\text{Total time}} = \frac{200 \text{ km}}{4 \text{ hr}} = 50$ km/hr.

Instantaneous :->

How fast object is moving

Inst. Speed = Rate of change in dist^n

$S = \left(\frac{dx}{dt}\right) = \text{dist}$

- Unit m/sec
→ Magnitude (Scale)

Inst. Velo = How fast & where

Inst. Velo = $\frac{d\vec{x}}{dt}$ = Rate of change of Position

Inst. Velocity = Inst Speed. direction

$\boxed{\text{Magnitude of velocity is Speed}}$

Unit m/s

Slope of Position time

Inst. Velocity.

DATE

Average :→

Avg. Speed = $\frac{\text{total dist}}{\text{total time}}$

→ Scalar
→ overall Idea of Journey
→ unit m/s

Avg Velo = $\left[\frac{\text{total disp}}{\text{total time}}\right]$

→ Vector
→ Unit m/s
→ overall Idea
→ with directly

$$\boxed{S_{avg} = \frac{\int s\,dt}{\int dt}}$$

$$V_{avg} = \frac{\int v\,dt}{\int dt}$$

Avg. Velo = Avg Speed × direction

→ Not always, only when objects moving without change in direction.

Average Velocity = Avg Speed → only correct when dir In not changing

Ins. Velocity = Ins. Speed → always because of an instant dist = disp

$$\boxed{\text{Equal time dist Interval} = \frac{2v_1 v_2}{v_1+v_2}}$$

Eq time Interval

$$V_{avg} = \frac{S_1 + S_2}{2}$$

$$S_{avg} = \frac{n}{\frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3}}$$

Speed

Inst. Speed

$S = \frac{d(x)}{dt}$

= Slope of dist / time graph is speed

$\int S dt = \int dx$

$\Rightarrow \underset{total}{dist}(x) = \int S_{IN} dt$

$$\boxed{\text{Avg. Speed} = \frac{\text{total dist}}{\text{total time}}}$$

$\frac{\int S_{Inst} dt}{\int dt} = S_{Avg}$

(Q) If $\boxed{S = 2t}$

Find avg. speed in 3 sec.

$S_{avg} = \frac{\int S_{ins} dt}{\int dt} = \frac{2\int_0^3 t\,dt}{\int_0^3 dt}$

$= \frac{\cancel{2}\,(t^2)_0^3 / \cancel{2}}{(t)_0^3} = \frac{9}{3}$

$= \boxed{3\,m/s}$

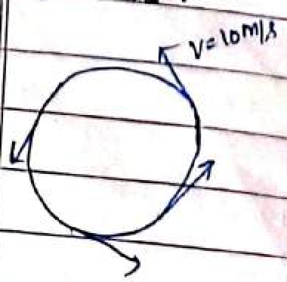
Uniform Speed.

Qu If Speed of object $S = 3t^2 + 2t$ then find Avg speed in 1st 2-sec.

① avg speed $= \frac{\text{total dist}}{\text{total time}}$

② Avg speed $= \frac{\int S dt}{\int dt}$

Q If speed is uniform (constant) hence how fast is constant.



$S_{avg} = \frac{\int S_{inst} dt}{\int dt} = \frac{S_{inst} \int dt}{\int dt}$

$\boxed{S_{avg} = S_{inst}}$

for const

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Velocity :-

Instan. Velocity

Avg velocity :

$$\frac{\text{Total disp}}{\text{total time}} = \frac{\int V_{ins}\, dt}{\int dt}$$

May or may not equal to Avg speed

Inst. Velocity

$V_{insta} = \frac{dx}{dt}$ ← Rate of change in position ← slop of position/time

How fast / direction

↳ direction of motion

dir^n of velocity

$$\int_{x_1}^{x_f} d\vec{x} = \int \vec{V}_{inst}\, dt$$

$$disp^m = \int \vec{v}_{inst}\, dt$$

Non Uniform :- → due to change in speed

→ due to change in direction

→ due to change in Both

$\boxed{\text{Uniform motion (velocity)}}$

Equal time interval

equal time Interval

S_{avg} Speed $= \frac{V_1 + V_2}{2}$

Equal dist diya hai to ye formula

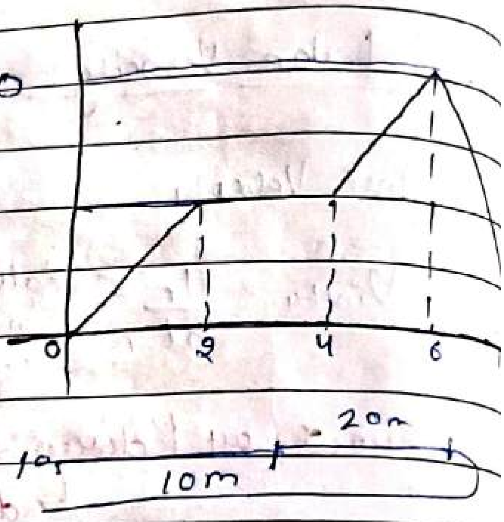
equal dist

S avg speed $= \frac{2 V_1 V_2}{V_1 + V_2}$

Que find avg Speed and avg velocity in 10 sec

avg velocity = $\frac{\text{total disp}}{\text{total time}} = 0$

Avg Speed = $\frac{\text{total dis}}{\text{total time}} = 40$



motion की दशा

Acceleration :- The rate of change in velocity w.r.t time is called acceleration.

V = 0 ⟶ t=0   V = 10 m/s ⟶ t = 1 s   V = 18 m/s ⟶ t = 3   V = 24 m/s ⟶ t = 4.5

$$a_{avg} = \frac{V_f - V_i}{\Delta t} = \frac{10-0}{1 s} = \boxed{a = 10 m/s^2}$$

Ex

V = 500, t = 0 ⟶ V = 502, t = 1s

$a = \frac{502 - 500}{2}$

$= 2 m/s$

Que 50 m/s, t = 0 ⟶ V = 48, t = 1 sec

$\vec{a} = \frac{48 - 50}{1} = -2 m/s$

retardation

acceleration is opposite to the velocity.

V = -48 m/s, t = 1 ⟵ V = -50 m/s, t = 0, $\vec{a}$ ⟶

$\vec{a} = \frac{-48 - (-50)}{1 s} = +2 m/s^2$

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In (A) we are safe

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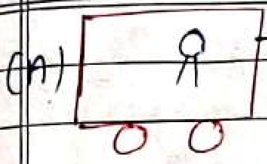
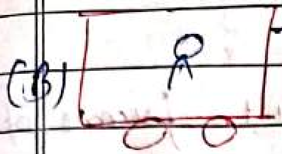
(A)  $\rightarrow V = 500\,m/s$ $v = 502\,m/s$ $504\,m/s$

$a = 2\,m/s^2$ $t = 1\,sec$ $t = 2\,sec$

(B)  $\rightarrow v = 2\,m/s$ $v = 502\,m/s$ $V = 1002\,m/s$ $V = 1502$

$a = 500\,m/s^2$ $t = 1\,sec$ $t = 2$ $t = 3\,sec$

Q (1)

$$\boxed{a_{inst} = \frac{d\vec{v}}{dt}}$$

= Slope of V-t graph

= unit m/s^2

= Vector quantity

= direcⁿ along change in Velocity

$$\vec{a}_{avg} = \frac{V_f - V_i}{\Delta t} = \frac{\Delta V}{\Delta t}$$

$$a_{avg} = \frac{\int_{t_1}^{t_2} a_{inst}\, dt}{\int_{t_1}^{t_2} dt}$$

If $V_{inst} = 0$ then accⁿ must be zero = False

If $(V = \text{const})$ then acc. must be zero $\rightarrow$ True

$$\vec{a} = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right)$$

Que (2)

$$\boxed{\vec{a} = \frac{d^2(\vec{x})}{dt}}$$

$$\vec{a} = \frac{d\vec{v}}{dt} \times \frac{dx}{dt}$$

Q (3)

$$\boxed{\vec{a} = \frac{dv}{dx} v^2}$$

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PAGE

$$\vec{a} = \left(\frac{dv}{dt}\right) = \left(\frac{d^2x}{dt^2}\right) = v\left(\frac{dv}{dx}\right)$$

$$\boxed{S_{avg} = \frac{2 S_1 S_2}{S_1 + S_2}}$$

Position :-> Location of object w.r.t frame of reference

dispm = Change in Position

dist = total path length

Vector Velocity → avg velocity = $\frac{\text{total disp}}{\text{total time}} = \frac{\int V_{inst}\, dt}{dt}$

→ Inst. Velocity = $\frac{dx}{dt}$

Scalar Speed → avg speed = $\frac{\text{total dist}}{\text{total time}}$

→ Inst. speed - ~~total time~~ Inst Velocity.

$\frac{d\vec{x}}{dt} = \vec{v}$

$\int d\vec{x} = \int \vec{v}\, dt$

Change in Position $= x_f - x_i = \text{disp} = \int_{t_1}^{t_2} \vec{v}\, dt$

$a = \frac{dv}{dt}$

$\int_{v_i}^{v_f} dv = \int a\, dt$

$\Delta v = \vec{V}_f - \vec{V}_i = \int a\, dt$ = Change in Velocity

dispm = Area of V/t graph

area of a/t graph

$\vec{x}$ position

$\boxed{\text{Velocity } (\vec{v})}$

acceleration ($\vec{a}$)

$\vec{v} = \frac{dx}{dt}$ (Slope of x-t graph)

$\vec{a}_{inst} = \frac{dv}{dt}$ (slope of v-t graph)

$\vec{a}_{inst} = v \cdot \frac{dv}{dx}$

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$disp = \int v\,dt$ $\qquad v_f - v_i = \Delta v = \int_{t_1}^{t_2} a\,dt = $ area of a/t graph

$\vec{x}$ (position) $\qquad \vec{v}$ (velocity) $\qquad$ accⁿ

slope $\frac{da}{dt}$

$\vec{v} = \frac{dx}{dt}$ (slope of x-t graph) $\qquad \vec{a} = \frac{dv}{dt}$

$\vec{a} = \frac{v\,dv}{dx}$

Acceleration = 0	Acceleration = Const	accⁿ = Variable
$\vec{a} = 0 = \frac{d\vec{v}}{dt}$	equation of motion variable = velo.	then accⁿ will be integration and diffⁿ
* Velocity = Const	$\vec{v}_f = \vec{v}_i + \vec{a}t$	diff and integral used
* Uniform motⁿ	$\vec{S} = \vec{u}t + \frac{1}{2}at^2$	
* $\vec{V}_{inst} = \vec{V}_{avg}$	$v_f^2 - v_i^2 = 2as$	
$\boxed{S = vt}$		
* $disp^m = \int v\,dt$	* Non Uniform motion	
* $\boxed{disp = V \times time}$		

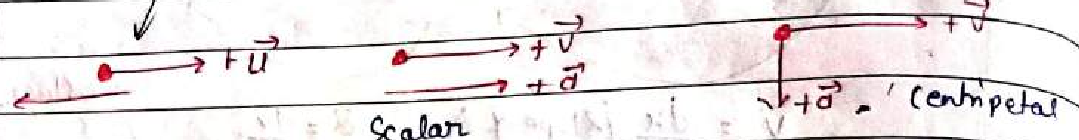
Velocity = Speed × direction

→ Change in speed

→ Change in dire

Change in both

Tangential acceleration.



Scalar

$\vec{F} = m\vec{a}$ (dirⁿ of accⁿ along force)

. 'centripetal = normal acceleration

Angle b/w $\vec{v}$ and $\vec{a}$ is $180°$

Angle b/w Velocity and acceleration is zero

Angle b/w $\vec{v}$ and $\vec{a}$ is $90°$

Speed → decreasing

Speed → Increasing

$\vec{a}.\vec{v} = 0$

$\vec{a}.\vec{v} = av\cos(180°)$
$= -ve$

$\vec{a}.\vec{v} = +ve$

Speed ↑

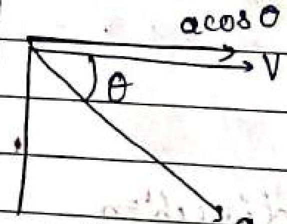
= at this Instant Speed remain Same

No change in dirⁿ at this Instant

No change in dire

dire will change

$a\cos\theta = a_T$ = accⁿ which is always velocity



θ = 180°

θ = 0°

$a_c = a\sin\theta$
(Perpendicular to Velocity)

Motion in Straight line
→ there may be U. turn due to tangential
→ Instantaneous dirⁿ change in not allow

$\boxed{a_c = 0}$ Centripetal accⁿ must be Zero.

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moving frame se body ko drop / release karne par frame ka velocity share ho jata hai but not acceleration.

$\vec{a} = \frac{d\vec{v}}{dt}$ = The rate of change in velocity w.r.t time

$\vec{a}_T = \frac{d\,[\vec{v}]}{dt}$ = The rate of change in magnitude of velocity w.r.t. time

diff

mag

$|\vec{a}| = \left|\frac{dv}{dt}\right|$ = The magnitude of the rate of change in velocity

mod

a

Constant acceleration :-

Position $x = t^3 + 2t + 5$ find velocity and accel^n

$\vec{V} = \frac{dx}{dt} = 3t^2 + 2$

$\vec{a} = \frac{d\vec{v}}{dt} = 6t$

Variable

Position $x = 5t^2 + 4t + 5$

$\vec{V} = \frac{dx}{dt}\ 10t + 4$

$\boxed{\vec{a} = 10}$

Const accel^n

$x = \frac{1}{t^2}$

$x = t^{-2}$

$v = \frac{dx}{dt} = -2t^{-3}$

$= \frac{-2}{t^3}$

Ans $x = A\sin(\omega t)$ = Variable acceleration

Qu $V = \sqrt{x}$ find accⁿ

$a = v\frac{dv}{dx} = \sqrt{x}\left(\frac{1}{2}x^{-1/2}\right)$

$a = \left(\frac{1}{2}\right) m/s^2$

$V = x^2$
$V = \frac{1}{x}$
$V = x^{3/2}$
always accⁿ Variable

a = const

$x \propto t^2$
$V \propto t^1$
$V \propto \sqrt{x}$

Qu Object is moving such that $V^2 = kx$ then find Position as a function of time.

(a) $x \propto t^3$
(b) $x \propto \sqrt{t}$
(c) $x \propto t^2$
(d) $x \propto \frac{1}{t}$

$V \propto \sqrt{x}$
$x \propto t^2$

$V^2 = Kx$
$V = \sqrt{Kx}$
$\frac{dx}{dt} = \sqrt{Kx}$
$dx = \sqrt{Kx}\,dt$
$\int \frac{dx}{\sqrt{Kx}} = \int dt$

Qu If Position $x = t^2 + 5t^3 + 6$ then find
(i) Initial acceleration — Initial Position
(ii) Initial Velocity
(iii) acceleration at $t = 2$ sec

$x(t=0) = 6m$

$x = t^2 + 5t^3 + 6$

$\boxed{v = \frac{dx}{dt} = 2t + 15t^2}$ $v(t=0) = 0$

$\boxed{a = \frac{dv}{dt} = 2 + 30t}$ $a(t=0) = 2\ m/s^2$

$a(t=2) = 2 + 30 \times 2$
$a = 62\ m/s^2$

In case of constant acceleration

$\vec{s} = \vec{u}t + \frac{1}{2} at^2$

$v^2 = u^2 + 2as$

$s = \left(\frac{\vec{u}+\vec{v}}{2}\right) t$

$$\boxed{V_{mid} = \sqrt{\frac{u^2+v^2}{2}}}$$

$\vec{V}_{avg} = \frac{\vec{u}+\vec{v}}{2}$

$\boxed{a = \text{const.}}$

$V = \vec{u} + at$

$s_{n^{th}} = u + \frac{a}{2}(2n-1)$

s (stopping distance) $= \frac{u^2}{2a}$

$\frac{s_1}{s_2} = \frac{u_1^2}{u_2^2}$

Ratio of displacement in t sec

$S_{1s} : S_{2s} : S_{3s} = 1 : 4 : 9 : 16$

$S_t : S_{2t} : S_{3t} = 1 : 4 : 9 : 16$

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Rest to Rest Motion :-

Object starts his motion from rest and const acceleration α for time t_1 after it retards with β and comes to at rest in t_2 time then find (i) V_{max} (ii) total disp

$u=0$ α = accⁿ, t_1 V_{max} $-\beta$ = retardation, t_2 $V=0$

2 sided of motion

$V = u + at$

$\boxed{V_{max} = \alpha t_1}$ — (i)

$v^2 - 0^2 = 2\alpha x_1$

(iii) $\boxed{V_{max}^2 = 2\alpha x_1}$

$V = u + at$

$0 = V_{max} - \beta t_2$

$\boxed{V_{max} = \beta t_2}$ — (ii)

$\boxed{\alpha t_1 = \beta t_2}$

$0 - V_m^2 = 2\beta x_2$

(IV) $V_m^2 = 2\beta x_2$

(III) = (IV)

$\boxed{\alpha x_1 = \beta x_2}$

$\boxed{\alpha t_1 = \beta t_2}$

rest to rest के लिए

Qu If total time of journey is T then find V_{max} in terms of 'α' and 'β'

$t_1 + t_2 = T$

Putting the value of t_1 & t_2 from eq ① & ②

$\frac{V_{max}}{\alpha} + \frac{V_{max}}{\beta} = T$

$V_{max}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = T$

$\boxed{V_{max} = \left(\frac{\alpha\beta}{\alpha+\beta}\right) T}$

$\boxed{S = \frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right) T^2}$

$\boxed{S = x_1 + x_2}$

unit / 01m

$V = at$

$\frac{a^2}{a}$ T

$\boxed{V = aT}$

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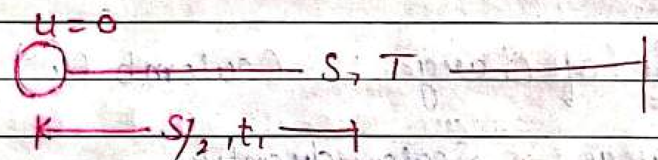
Rest to Rest

- $\alpha t_1 = \beta t_2$
- $\alpha x_1 = \beta x_2$
- $V_{max} = \left(\frac{\alpha\beta}{\alpha+\beta}\right)T$ Total time
- Total disp $S = \frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right)T^2$

Body starts his motion from rest and const acceleration it takes T time in S dispm then find time taken to cover half dispm S/2 in terms of 'T'



for Comp Journey

$S = \frac{1}{2}aT^2$ —— ①

for 1st half journey

$\frac{S}{2} = \frac{1}{2}at_1^2$ —— ⑪

$$\frac{\cancel{S} = \cancel{\frac{1}{2}}\cancel{a}T^2}{\frac{\cancel{S}}{2} = \cancel{\frac{1}{2}}\cancel{a}t_1^2} = 2 = \frac{T^2}{t_1^2}$$

$t_1 = \frac{T}{\sqrt{2}}$

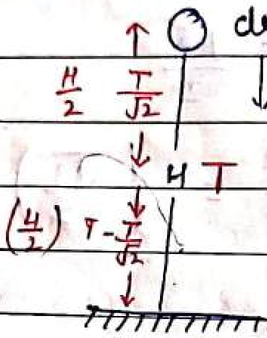
Motion Under gravity :→

- motion with Constant acceleration
- acceleration const = g downward
- air friction = 0

= all equation of motion is valid.

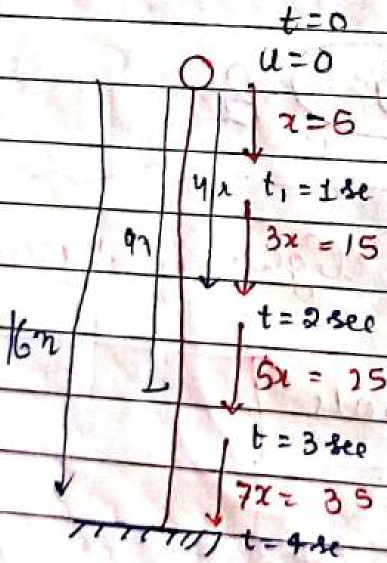
→ drop from Height ('H')

Non Uniform



drop ($u=0$)

$\downarrow a = g$ (const) $= 10\,m/s^2$



$t=0$

$u=0$, $a=+g$ (upward)

$x = \frac{1}{2} g(1)^2 = \left(\frac{g}{2}\right) = 5\,m$

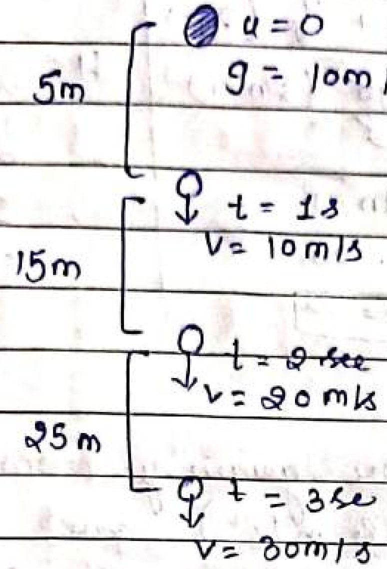
$y = \frac{g}{2}[2\times 2 - 1] = \frac{3g}{2} = 15m$

$\Rightarrow S_{1se} : S_{2^{nd}se} : S_{3^{rd}se} : S_{4th\,se} = 7, 3x, 5x, 7x$

$\Rightarrow S_{1s} : S_{2sec} : S_{3sec} : S_{4sec} =$

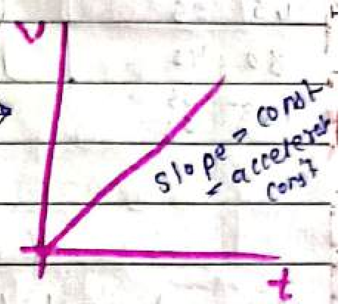
$x\ 4x : 9x : 16x : 25x$

Qu Object is droped then Velocity, disp^n after time 't' Draw graph.

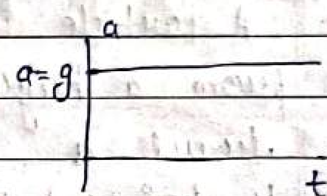


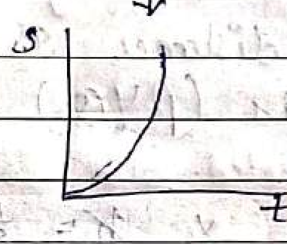
$v = u + at$

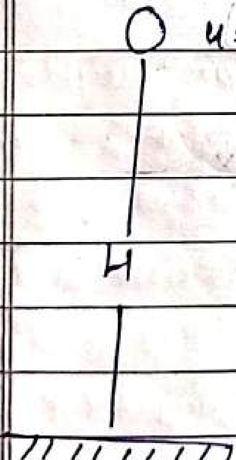
$\boxed{v = gt}$ →



$\boxed{s = \frac{1}{2} g t^2}$







Time of fligh =

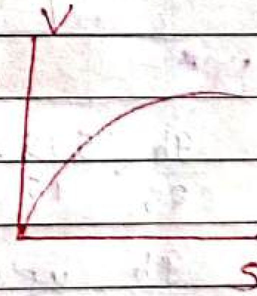
$T = \sqrt{\frac{2H}{g}}$

3^{rd} eq of motion

$v^2 - \cancel{u^2} = 2as$

$v^2 = 2gH$

$v = \sqrt{2gH}$ → Velocity at ground.



Que Object is draped and distance travelled in last 1 Sec is equal to 1st 3-sec then find height from Ground from where Object is droped

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$t = 3$ sec $\left\{ \right.$ $S = 5t^2$

$5 \times (3)^2 = 45$ m

total time of flight $= 50$

n-sec	n^{th} sec	H
5	5	
20	15	
45	25	
80	45	
125	55, 65	

1sec n^{th} sec

$S_{n^{th}} = u + \frac{a}{2}(2n-1)$

$45 = \frac{\cancel{10}^5}{\cancel{2}}(2n-1)$

$9 = 2n - 1$

$10 = 2n$

$\boxed{n = 5}$

$H = \frac{1}{2} at^2$

$\frac{1}{2} 10(5)^2$

5×25

$= 125$ m

Que A Particle is dropped under gravity from rest from a height h ($g = 9.8$ m/s^2) and travel a distance $9h/25$ in the last second, the height h is (PYQ)

First Method.

(T max) H

$h - \frac{9h}{25} = \frac{16h}{25}$ (T-1)

$9h/25$

$\cancel{h} = \frac{1}{\cancel{2}} \cancel{g}T^2$ — (1) ×

$\frac{9\cancel{h}}{25} = \frac{\cancel{g}}{\cancel{2}}(2T-1)$

$\frac{25}{9} = \frac{T^2}{2T-1}$ ×

MR

$\frac{9h}{25} = 5, 15, 25, 35, 45, 55$

$\frac{9h}{25} = 45$

$\boxed{h = 125}$

$\cancel{h} = \frac{1}{\cancel{2}} \cancel{g}T^2$

$\frac{16\cancel{h}}{25} = \frac{1}{\cancel{2}}\cancel{g}(T-1)^2$

$\frac{25}{16} = \frac{T^2}{(T-1)^2}$

$\frac{T}{T-1} = \sqrt{\frac{25}{16}} \Rightarrow \frac{T}{T-1} = \frac{5}{4}$

$4T = 5T - 5$

$5 (5T - 4T)$

$T = 5$ sec

your writing partner

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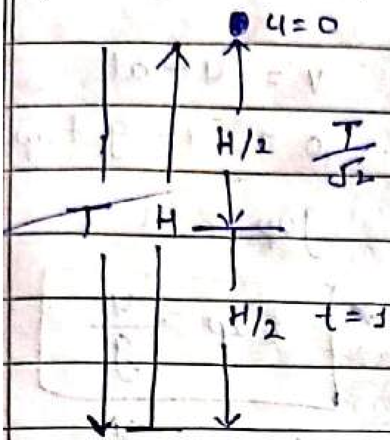
Object is dropped then it moves 2nd half distance in last 1 sec of motion, then find time of flight ? IIT 2004.



$$T - \frac{T}{\sqrt{2}} = 1$$

$$T\left(1 - \frac{1}{\sqrt{2}}\right) = 1$$

$$T\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right) = 1$$

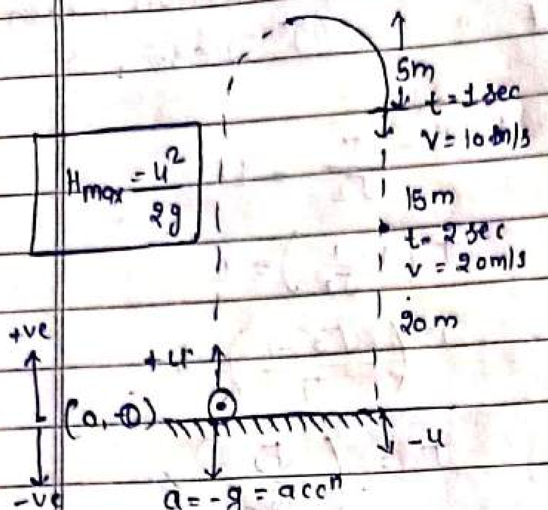
$$T = \left(\frac{\sqrt{2}}{\sqrt{2}-1}\right) \text{sec}$$

$$T = \frac{\sqrt{2}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{\sqrt{2}(\sqrt{2}+1)}{2-1}$$

$$= \frac{2+\sqrt{2}}{2-1}$$

$$= \boxed{2+\sqrt{2}}$$

Motion under gravity from ground to ground



Time of flight T_f

$v = u + at$

$0 = u - g\,t_{up}$

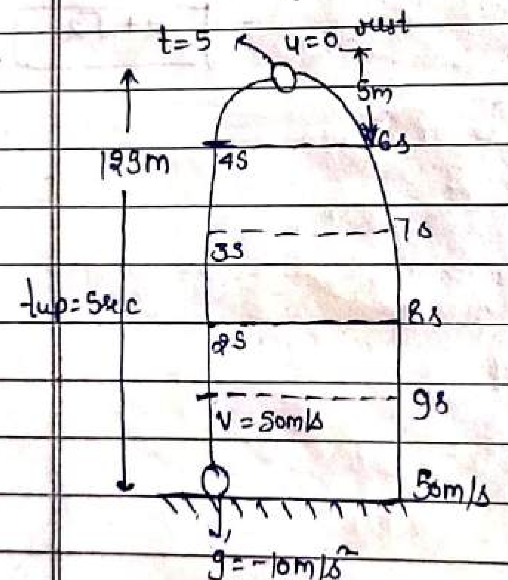
$g\,t_{up} = u$

$\boxed{t_{up} = \frac{u}{g}}$

$t_{down} = \frac{u}{g}$

$\boxed{T_f = \frac{2u}{g}}$

Ball is projected with 50 m/s then find



(i) Time of flight $= \frac{2u}{g} = \frac{2 \times 50}{10} = 10$ sec

(ii) Max^m Height =

(iii) disp^m in 6 sec

(iv) disp^m in 6 sec

(v) Height at 7 sec

(vi) Avg Velocity in 8 sec

(vii) Avg Speed in 8 sec

Avg velocity $= \frac{disp}{time} = \frac{80}{8}$

Avg speed $= \frac{125 + 45}{8}$

$\frac{170}{8}$ m/s

(ii) $H_{max} = \frac{u^2}{2g} = \frac{(50)^2}{2 \times 10} = \frac{2500}{2 \times 10}$

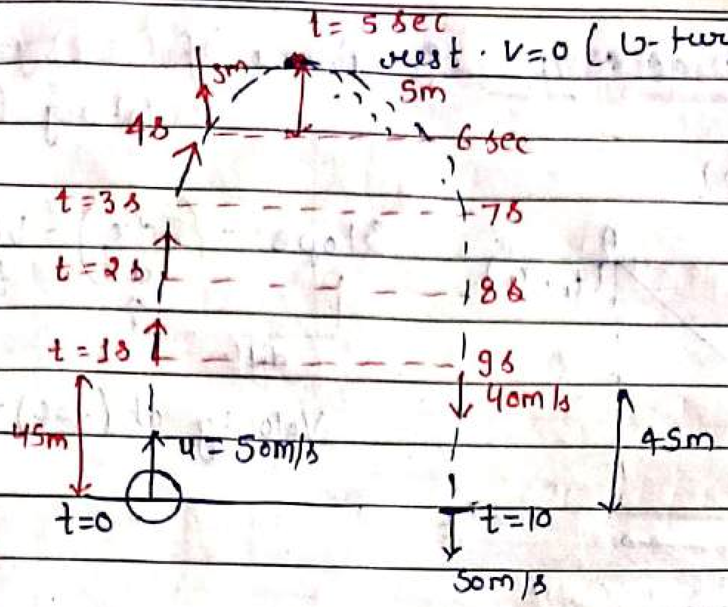
$= \boxed{125\ m}$

(iii) $S = ut + \frac{1}{2}at^2$

$S = 50 \times 6 - \frac{1}{2} 10 (6)^2$

$S = 300 - 5 \times 36$, $S = 300 - 180 \Rightarrow \boxed{S = 120\ m}$

(iv) distance = 130 m Ans



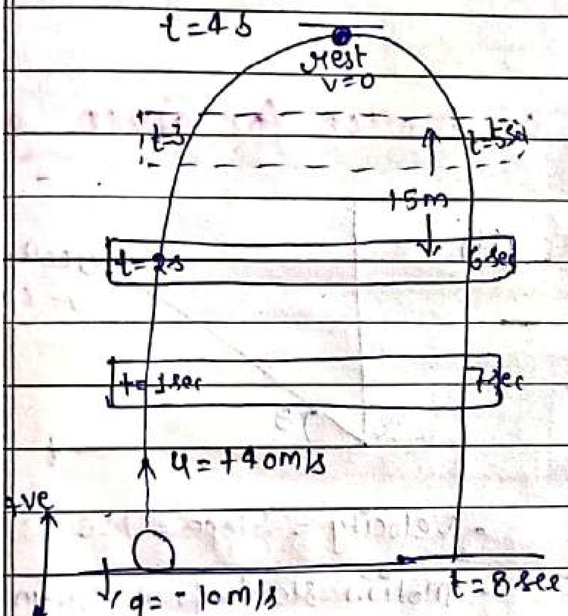
disp^m in 1st sec

$s = ut + \frac{1}{2}at^2$

$50 \times 1 - \frac{1}{2} \times 10(1)^2$

$50 - 5$

$\boxed{45\ m}$



$T_f = 4s + 4s = 8$ sec

$H_{max} = \frac{u^2}{2g} = 80m$

disp^m in 6th sec = 15 m

$S_{n^{th}} = u + \frac{a}{2}(2n-1)$

$40 - \frac{10}{2}(6 \times 2 - 1)$

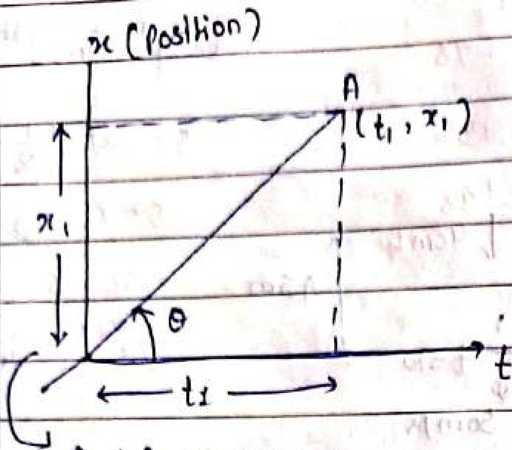
$40 - 5 \times 11$

$40 - 55 = -15\ m$

Average speed in 7-sec $= \frac{80 + 45}{7}$

$\frac{125}{7}$ m/s

Position time graph :- Area of (x-t) graph = Integ (x-t) = nil



Origin पर स्थिति zero

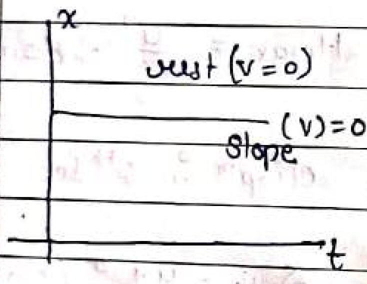
Slope $= \left(\frac{dx}{dt}\right) =$ Velocity $= \tan\theta$

diff.

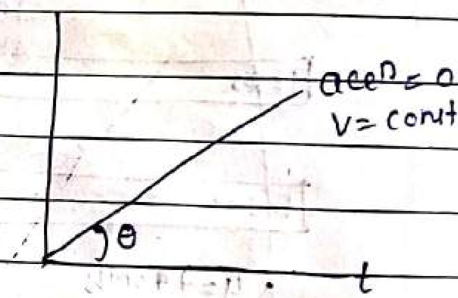
Velocity at $(t=0) =$??

Comment nature of motion for given graph

(i)

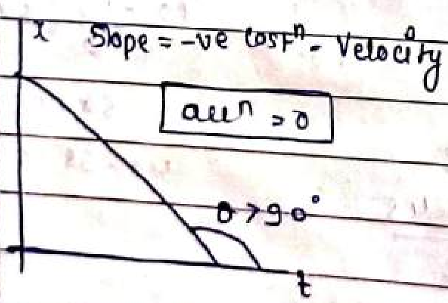


(ii)

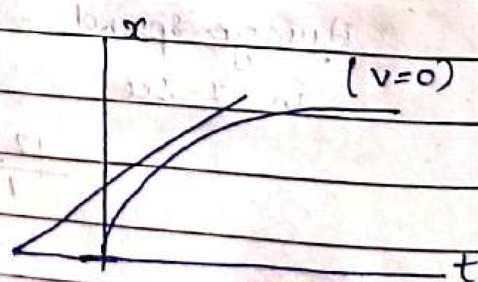


- Velocity = Slope = +ve & Constⁿ
- Motion starts from origin and Constⁿ velocity.

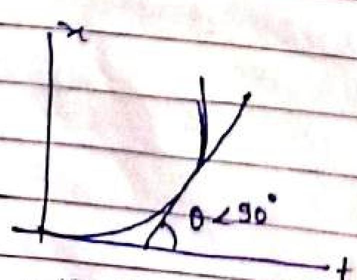
(iii)



(v)



(iv)



motⁿ start from rest (v=0) at (t=0) Increasing Velocity

Velocity = +ve ↓

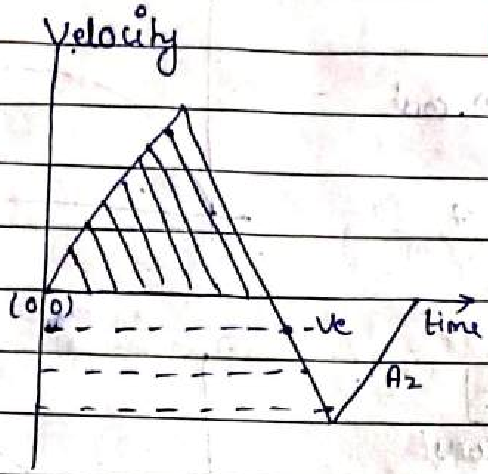
Speed ↓

retardation

accⁿ is opposite to Velocity

$\boxed{a = -ve}$

Velocity - time graph



at $t = 0$
Velocity is zero

Velocity = { Slope ✗, Area ✗, y th coordinate पर velocity.

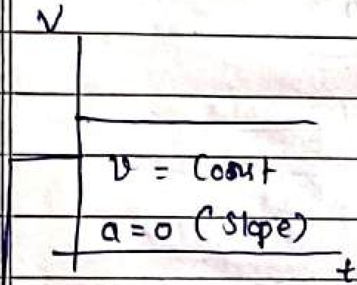
* Slope $= \frac{dv}{dt} = \vec{a}_{inst}$

Area Under (v-t) graph
$= \int v\,dt = disp^m$

$disp^m = A_1 - A_2$
$disp^m = A_1 + A_2$

Comment nature of motion for given graph.

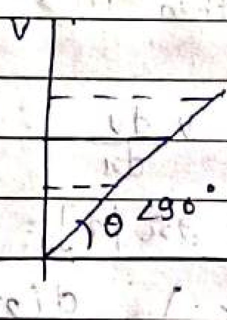
(i)



v = Const
$a = 0$ (Slope)

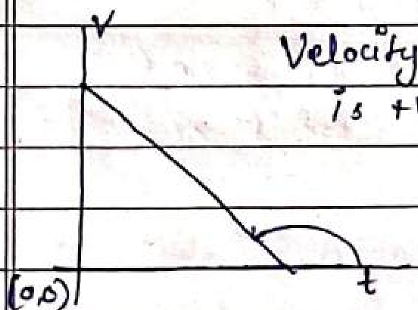
→ uniform motion

(ii)



Slope = +ve Const
acc^n = +ve, $Cost^n$
Velocity in Increasing.

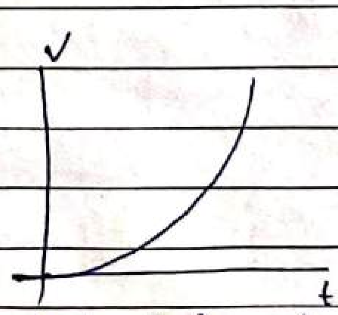
(iii)



Velocity is decreasing
is +ve

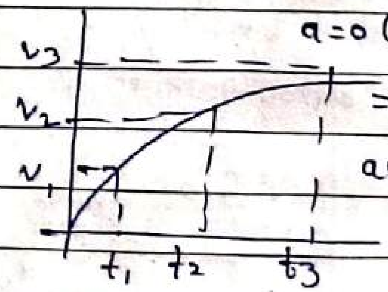
Slope = $-Cost^n$
↓
acc^n = -ve Const

(iv)



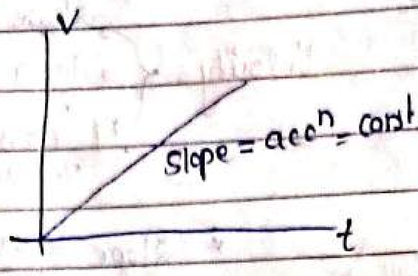
velocity is increasing in Position
Slope (acc^n) = Increasing

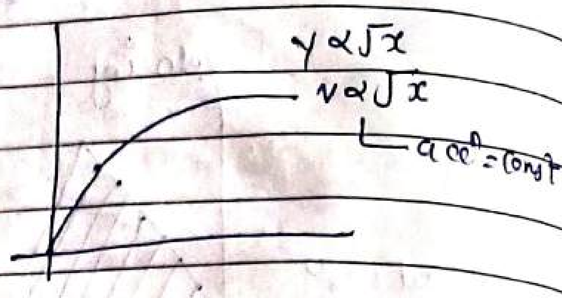
(v)

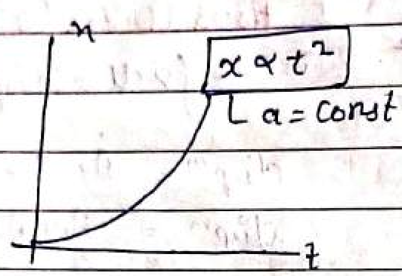


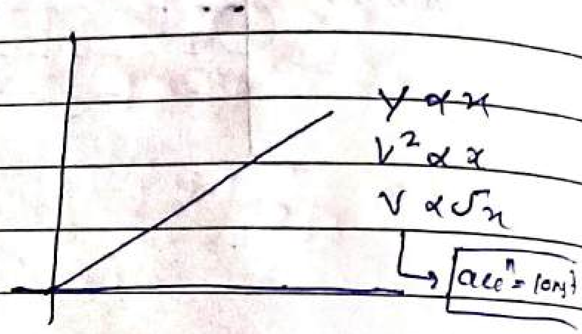
$a = 0$ (slope = 0)
= V is Increasing
acc^n is decreasing

In Which graph acceleration is Constant









Que If $v = 2x$ then find acc^n

Solⁿ $acc^n = v\dfrac{dv}{dx}$

$= 2x\,[2] = 4x$

$\left(\dfrac{dv}{dx}\right) = \dfrac{d(2x)}{dx} = 2 \times 2 = 2$

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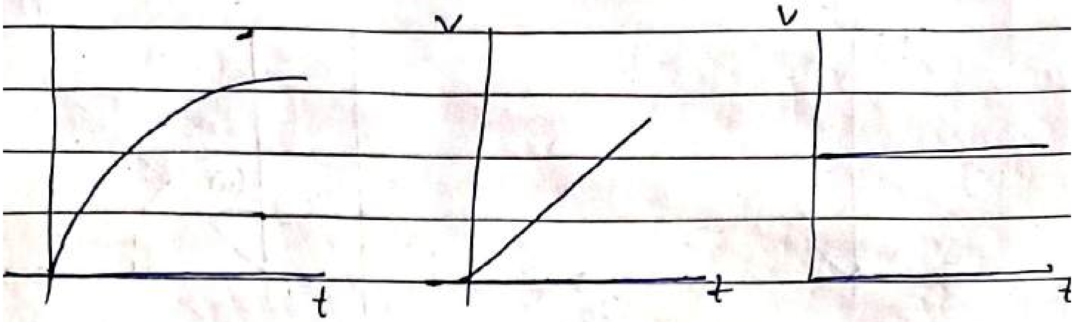
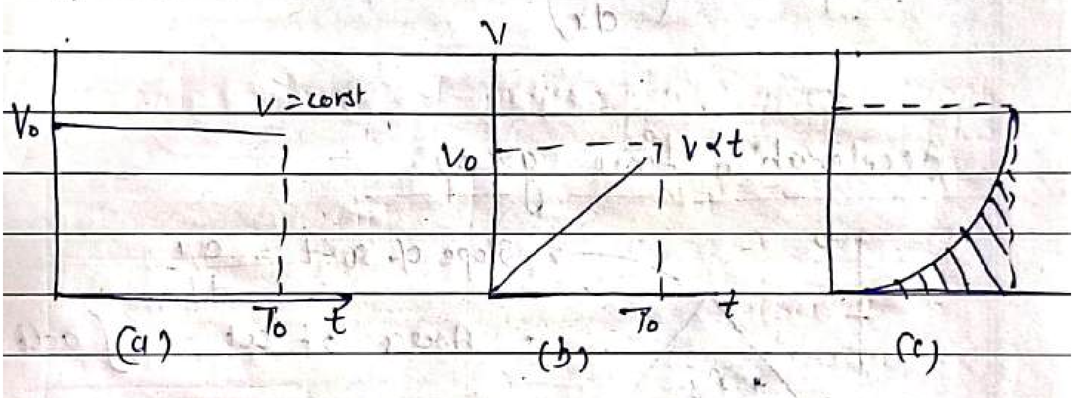
In which graph motion is Uniform



Uniform motion = Velocity Const

In which graph Average Velocity is maximum b/w 0 to T_0

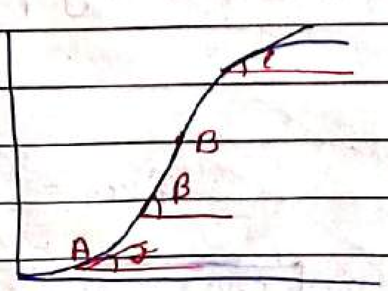


$$\text{Avg velocity} = \frac{\text{disp}^m}{\text{time}} = \frac{(\text{Area}) \text{ Max}}{\text{time}} \rightarrow$$

$$\text{Avg acc}^n = \frac{v_f - v_i}{\Delta t}$$

(i) Velocity is maximum at

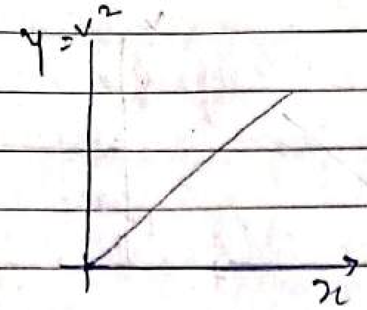
(ii) Nature of mot^n → uniform mot^n
→ Uniform acc^n
→ Non uniform acc^n mo

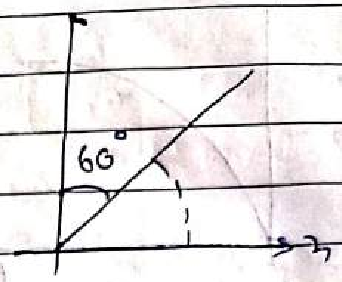


Ans Velocity is max^m at 'B'

Graph b/w (v^2) and (x)

Find accn





Slope of $(v^2)-(x) = \frac{dv^2}{dx}$

$= \frac{dv^2}{dv} \times \frac{dv}{dx}$

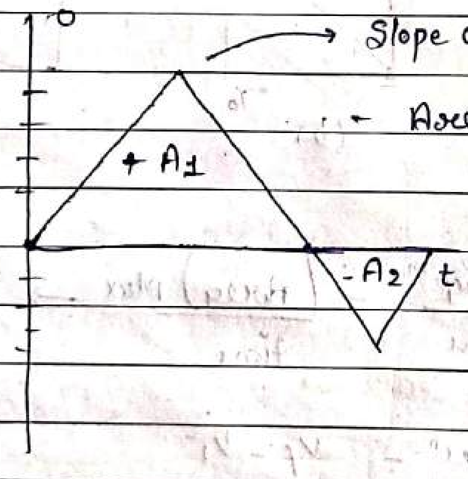
Slope $= 2\left(v\frac{dv}{dx}\right) = 2 \times acc^n$

Slope $= 2a$

$\tan 30° = 2a$

$a = \frac{1}{2\sqrt{3}}\ m/s^2$

Acceleration time graph :-



→ Slope of $a-t = \frac{da}{dt}$

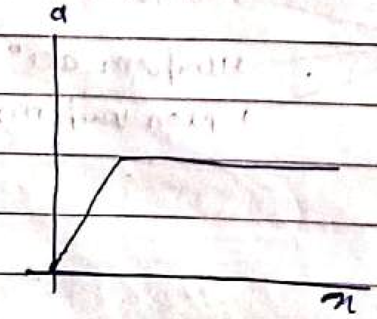
→ Area of $a t$ → $\int a\,dt$

Δv = Area

$$\boxed{\vec{V_f} - \vec{V_i} - \text{Area of at grap}}$$

$\vec{V_f} - \vec{V_i} = A_1 - A_2$

a–x graph →



area of $a-x$ graph $= \int a\,dx$

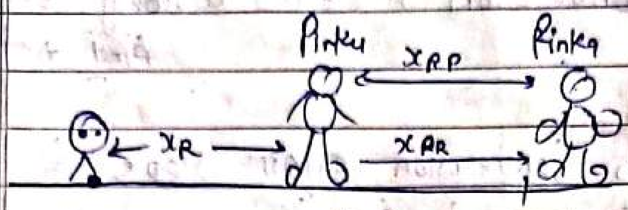
$= \int v\frac{dv}{\cancel{dx}}\,\cancel{dx}$

Area $= \int_u^v v\,dv = \left(\frac{v^2}{2}\right)_u^v$

$$\boxed{\text{Area of } a-x \text{ grap} = \frac{v^2-u^2}{2}}$$

DATE

Relative Motion in 1-D



$$\boxed{x_{RP} = x_{Rg} - x_{Pg}}$$

diff^n w.r.t. time

$$\vec{V}_{RP} = \vec{V}_{Rg} - \vec{V}_{Pg}$$

x_{Rg} = Position of Ramlal from origin w

x_{Pg} = Position of Pinky (w.r.t ground)

x_{RP} = Position of Ramlal with respect to Pinky

$$\boxed{\vec{x}_{PR} = \vec{x}_{Pg} - \vec{x}_{Rg}}$$

Pinky observer

$$\vec{V}_{PR} = \vec{V}_{Pg} - \vec{V}_{Rg}$$

$$\vec{x}_{RP} = -\vec{x}_{PR}$$

diff wrt time

$$V_{RP} = -\vec{V}_{PR}$$

$$V_{AB} = \vec{V}_A + (-\vec{V}_B)$$

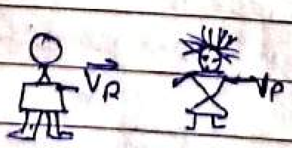
$$\vec{a}_{AB} = \vec{a}_{ag} - \vec{a}_{Bg}$$

$$\vec{a}_{BA} = \vec{a}_B - \vec{a}_A$$

Case- 1

(i) $\vec{V}_R = \vec{V}_P$

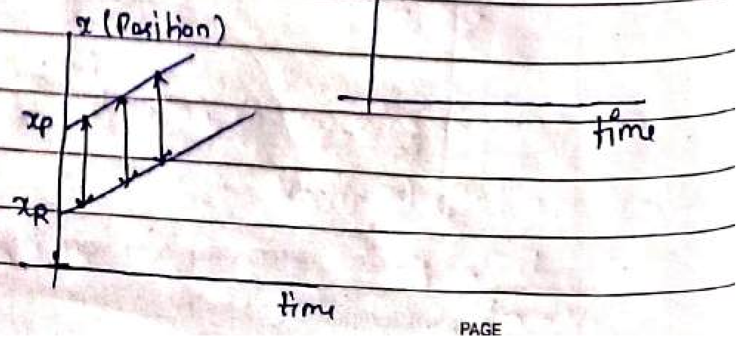
$\vec{V}_{PR} = \vec{V}_P - \vec{V}_R = 0$



x_{PR} = Position of Pinky w.r.t Ramlal

$$\boxed{V_{RR} = V_R - V_R = 0}$$

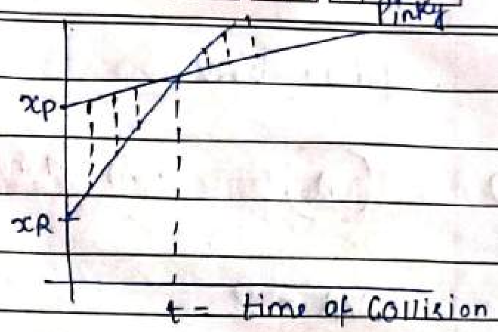
Observer always assume to be at rest



PAGE

DATE

Case-2

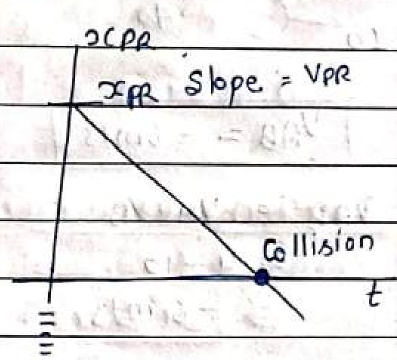


(ii) $\vec{V}_R > \vec{V}_P$

(#) $V_{PR} = \vec{V}_P - \vec{V}_R = -ve$

↳ velocity Pinky w.r.t Ramlal = -ve

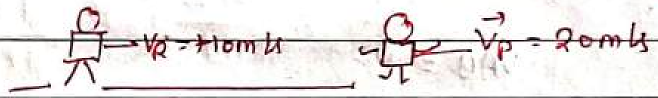
$\vec{V}_{RP} = \vec{V}_R - \vec{V}_P = +ve$



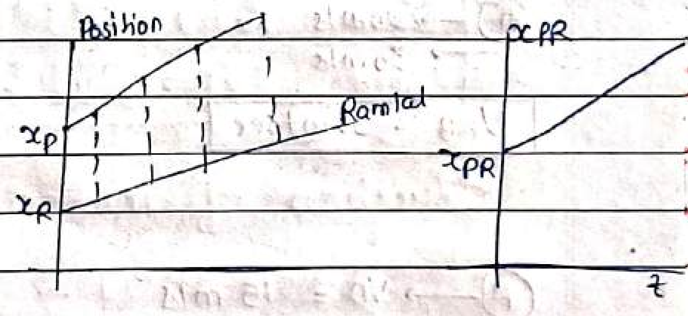
(iii) $\vec{V}_R < \vec{V}_P$

$\vec{V}_{PR} = \vec{V}_P - \vec{V}_R = +ve$

↑ observe

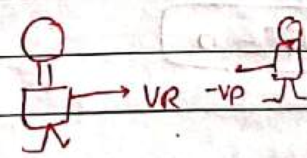


$\vec{V}_{RP} = V_R - V_P = -ve$

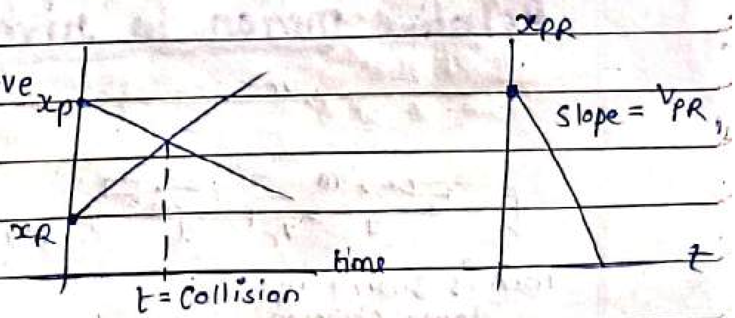


$\vec{V}_{PR} = \vec{V}_P - \vec{V}_R$

$= -ve$



$V_{RP} = \vec{V}_R - \vec{V}_P = +ve$



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find V_{AV}

① (A) $\longrightarrow V_A = 4\,m/s$ (B) $\longrightarrow 10\,m/s$

$V_B = ?$

$V_{BA} = V_B - V_A$
$= 10 - 4$
$= 6\,m/s$

$10 \longleftarrow \bigcirc \longrightarrow 4$

$\boxed{V_{AB} = -6\,m/s}$

$V_{AB} = V_A - V_B$
$= 4 - 10$
$= -6\,m/s$

(A) $\longrightarrow V_A = 20\,m/s$ $\longleftarrow$ (B) $V_B = -30\,m/s$

$V_{AB} = ?$

$V_{BA} = -50\,m/s$

(A) $\longrightarrow 20\,m/s$
$\longrightarrow 30\,m/s$

$30 \longleftarrow$ (B)
20

$\boxed{V_{AB} = 50\,m/sec}$

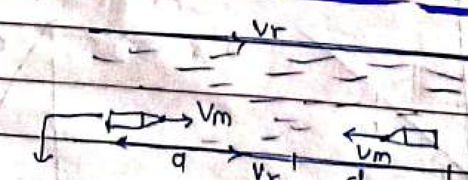
(A) $\longrightarrow V_A = 15\,m/s$ (B) $\longrightarrow 54\,Km/hr$

$\cancel{54}^{3} \times \frac{5}{\cancel{18}} = 15\,m/s$

$V_{AB} = ?$

$\boxed{V_{AB} = 15 - 15 = 0}$

Relative motion in river (one-dimension)



Man is swimming in down stream

man is swiming in upstream

V_{man} का effective $= V_m + V_r$

V_{man} का effective $= V_m - V_r$

$t = \frac{d}{V_m + V_r}$

$t = \frac{d}{V_m - V_r}$

DATE

Find time when they will meet again

A → $u_A = 4$ m/s
→ $a_A = 2a$ m/s²

B → $u_B = 24$ m/s
→ $a_B = a$ m/s²

$\boxed{s = ut + \frac{1}{2}at^2}$

$0 = ut - \frac{1}{2}at^2$

$u\cancel{t} = \frac{1}{2}at^{\cancel{2}}$

$\left(\frac{24}{a}\right) = t$

Relative motion in motion Under gravity

⇒ motion of A and B is non-uniform with constant acceleration

$a_A = g$ $a_B = g$

$\vec{u}_{BA} = \vec{v}_B - \vec{v}_A$

$\vec{a}_{BA} = a_B - a_A$

$g - g = 0$

$a_{BA} = 0$

→ Relative accn is zero
relative Velocity const^n
relative Mot^n uniform

$t = 0$
A: $a = 10$ m/s², $v_A = 10$ m/s
B: $u = 30$ m/s, $a = g = 10$ m/s

⇒ $t = 0$ $V_{BA} = 20$ m/s

$t = 1$ sec
$v = 20$ m/s $v = 40$ m/s
$t = 1$ $V_{BA} = 20$ m/s

$t = 2$ sec
$v = u + at$
$10 + 10 \times 2 = 30$ m/s $v = 50$ m/s
$t = 2$ sec $V_{BA} = 20$ m/s

$t = 3$ sec
$v = 40$ m/s $v = 60$ m/s $t = 3$ sec $V_{BA} = 20$ m/s

Motion in a Plane

DATE

2-D motion

Vector → Component of vector (तोड़ना)

Vector → Magnitude of vector (जोड़ना)

2D motion = [1-D] x-axis + [1-D] y-axis

Position vector in plane :-

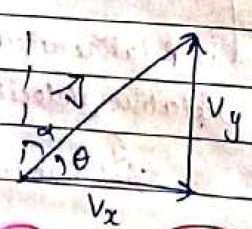
→ $\boxed{\vec{r} = x\hat{i} + y\hat{j}}$

diff w.r.t time

$$\frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$$\boxed{\begin{aligned} * \ \vec{v} &= \vec{V}_x \hat{i} + \vec{V}_y \hat{j} \\ * \ \text{Speed} &= |\vec{v}| = \sqrt{V_x^2 + V_y^2} \end{aligned}}$$

direction of motion



$\tan\theta = \frac{V_y}{V_x}$

dirⁿ of motⁿ from x axis

$\boxed{\vec{v} = \vec{V}_x \hat{i} + \vec{V}_y \hat{j}}$ $|\vec{v}| = \sqrt{v_x^2 + v_y^2}$

diff w.r.t time

$\boxed{\vec{a} = \vec{a}_x \hat{i} + \vec{a}_y \hat{j}}$

$|\vec{a}| = \sqrt{a_x^2 + a_y^2}$

$\boxed{\text{dispⁿ} = \vec{r}_f - \vec{r}_i}$

$\text{dispⁿ} = (x_f - x_i)\hat{i} + (y_f - y_i)\hat{j}$

DATE

Equation of motion in plane - for Const. accl^n

u_x v_x a_x

u_y v_y a_y

$$\boxed{\begin{aligned} \vec{V_x} &= \vec{u_x} + a_x t \\ \vec{V_y} &= \vec{u_y} + a_y t \end{aligned}}$$

$$\boxed{\begin{aligned} x &= u_x t + \tfrac{1}{2} a_x t^2 \\ y &= u_y t + \tfrac{1}{2} a_y t^2 \end{aligned}}$$

Qu If initial velocity of object $\vec{u} = 3\hat{i} + 4\hat{j}$ and after some time its velocity $v = 4\hat{i} + 3\hat{j}$ then find

(i) magnitude of change in velocity

(ii) Change in magnitude of velocity.

(i) $\vec{u} = 3\hat{i} + 4\hat{j}$

$\vec{v} = 4\hat{i} + 3\hat{j}$

$\Delta\vec{v} = \vec{v} - \vec{u} =$

$= (4\hat{i} - 3\hat{j}) - (3\hat{i} + 4\hat{j})$

$= \hat{i} - \hat{j}$

$|\Delta\vec{v}| = \sqrt{2}$

(ii) $|\vec{v}| = \sqrt{(4)^2 + (3)^2} = 5$

$|\vec{u}| = 5$

$|\vec{v}| - |\vec{u}| = 0$

Qu Object is moving with velocity $\vec{V} = 3\sin(\omega t)\hat{i} + 3\cos(\omega t)\hat{j}$ then find distance moved in 2 sec

distance = speed × time   variable of integration

if const

$\boxed{\vec{v} = 3\sin(\omega t)\hat{i} + 3\cos(\omega t)\hat{j}}$

v-r (m) speed variable x

$\underbrace{|\vec{V}|}_{\text{speed}} = \sqrt{3^2(\sin^2\omega t + \cos^2\omega t)} = 3$

dist = $3 \times 2 = 6$ m Ans

Equation of Trajectory :-

→ Path followed by Object

→ relation b/w 'x' and 'y' Position

↓

reln b/w y and x is called equation of tracetor

Que Position of object at time 't' $\vec{r} = 2t\hat{i} + 4t^2\hat{j}$. then find equation of trajectory.

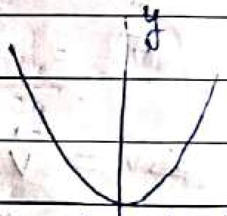
$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\boxed{\vec{r} = 2t\hat{i} + 4t^2\hat{j}}$$

$x = 2t$ | $y = 4t^2$

$t = \frac{x}{2}$ | $y = 4\left(\frac{x}{2}\right)^2$

$y = \frac{4x^2}{4} = y = x^2$



Parabolic Path

Que $\vec{u} = 2\hat{i} + 4\hat{j}$

$\vec{a} = -2\hat{j}$ find speed after $t = 2$ sec

x- axis

$a_x = 0$

$\vec{v_x} = \vec{u_x} = 2\hat{i}$

$\boxed{\vec{v} = 2\hat{i} + 0}$

Speed $= 2m/s$

y- axis.

$v_y = u_y + a_y t$

$= 4 - 2 \times 2$

$v_y = 0$

$y = mx + c$ = Straight line

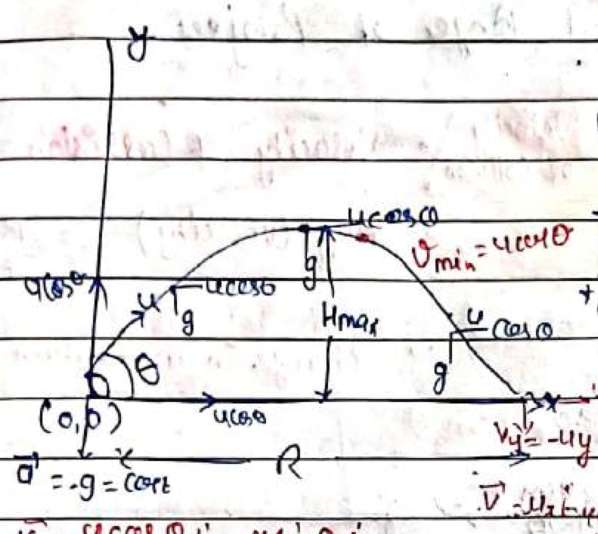
$y = x^2$, $y = \sqrt{x}$ } Parabola

$x^2 + y^2 = R^2$ = Circle

$\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$ ellipse

Projectile Motion

Projectile motion is 2-D non uniform motion with uniform acceleration.



x-axis	y-axis
$u_x = u\cos\theta$	$u_y = u\sin\theta$
$a_x = 0$	$a_y = -g$
$\vec{v}_x = u\cos\theta$	$v_y = (u\sin\theta - gt)\hat{j}$
$\boxed{\text{uniform mot}^n}$ in x axis	$\vec{s} = u_y t + \frac{1}{2}a_y t^2$
$x = u\cos\theta\, t$	Non. uniform
$x = u_x t$	

$\vec{a} = -g = \text{const}$

$\vec{v} = u\cos\theta\,\hat{i} - u\sin\theta\,\hat{j}$

Qu

$\vec{u} = 2\hat{i} + 4\hat{j}$

$\vec{a} = -2\hat{j}$ find speed after $t = 2$ sec

$a_x = 0$

$\vec{v}_x = \vec{u}_x = 2\hat{i}$

$\boxed{\vec{V} = 2\hat{i} + 0}$

Speed = 2 m/s

$v_y = u_y + a_y t$

$= 4 - 2\times 2$

$v_y = 0$

$\boxed{\text{disp}^t \text{ in } t = 3 \text{ sec}}$

$x = u_x t + \frac{1}{2}a_x t^2$

$x = 2\times 3 = 6\hat{i}$

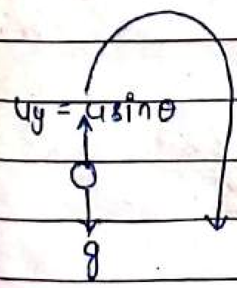
$y = u_y t + \frac{1}{2}a_y t^2$

$4\times 3 - \frac{1}{2}\times 2(3)^2$

$12 - 9 = \boxed{3\,m\,\hat{j}}$

Time of Flight

Consider mot^n y-axis only



$$\boxed{T = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}}$$

$$\boxed{R = \frac{u^2\sin 2\theta}{g}}$$

$$\boxed{H_{max} = \frac{u_y^2}{2g} = \frac{u^2 \sin^2\theta}{2g}}$$

Ball is projected with $\vec{u} = 3\hat{i} + 4\hat{j}$ then find H_{max}, T_f, R and Angle of Project

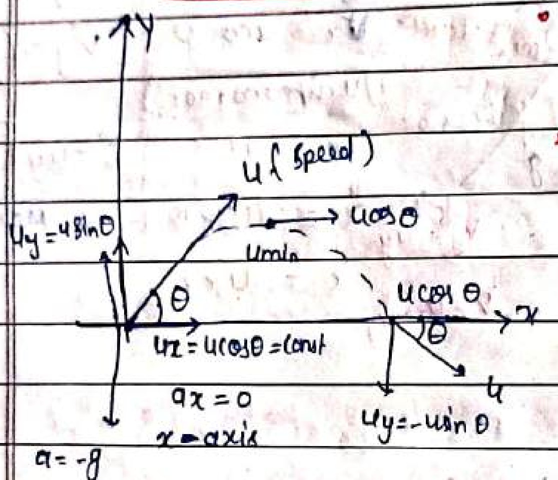
Velocity of Projection $\neq$ Velocity of Collision

- Velocity of Collision $= u\cos\theta\,\hat{i} - u_y\,\hat{j}$
- Avg: (velocity) $= \frac{R}{T_f} = \frac{u_x T_f}{T_f} = u_x = u\cos\theta$

* Change in momentum $= \Delta \vec{P} = \vec{P_f} - \vec{P_i}$

$m[\vec{v_f} - \vec{u_i}]$

$= m[-u_y\hat{i} - u_y\hat{j}]$

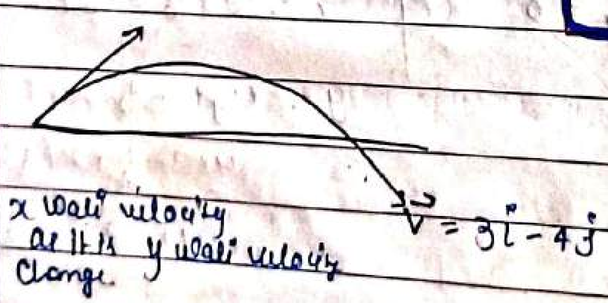


$$T_f = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}$$

$$H_{max} = \frac{u_y^2}{2g} = \frac{u^2\sin^2\theta}{2g}$$

$$R = u_x T_f = \frac{2u_x u_y}{g} = \frac{u^2\sin 2\theta}{g}$$

Que



Que find rate of change in momentum at half of max Height ?

DATE

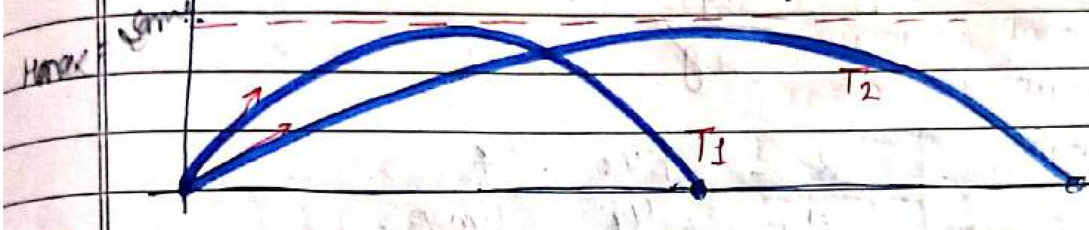
Q. Path of two Projected Particle is given then Compaire time of flight ?

$H_1 = H_2$

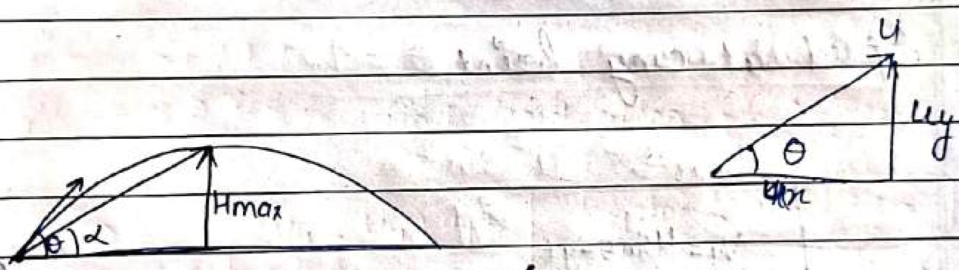
$H_{max} \frac{u_{y_2}}{2g}$ $H \frac{u_{y_2}^2}{2g}$ Vertical velocity same

time of flight = same

$\boxed{T_1 = T_2}$



Elevation angle of max^m height from point of Projection.



$$\tan\alpha = \frac{H_{max}}{R/2} = \frac{\frac{u_y^2}{2g}}{\frac{2u_x u_y}{2g}} = \frac{u_y}{4u_x} = \frac{\tan\theta}{2}$$

$$\tan\alpha = \frac{\tan\theta}{2}$$

$$\boxed{\alpha = \tan^{-1}\left(\frac{\tan\theta}{2}\right)}$$

$$\boxed{R_{max} = \frac{u^2}{g}, \quad \theta = 45^\circ}$$

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DATE

P.M.A

* Relation Between Horizontal Range and Maximum Height

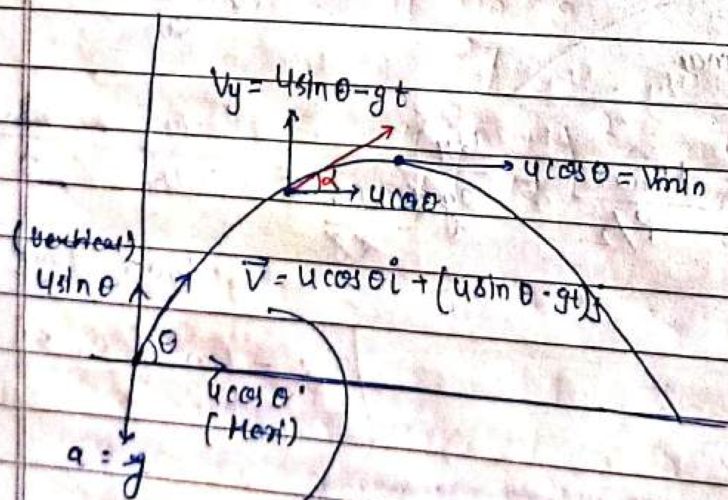
$$R = \frac{2 u_x u_y}{g}$$

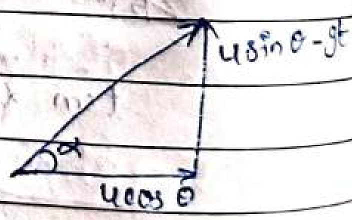
$$H = \frac{u_y^2}{2g}$$

$$\frac{R}{H} = \frac{4 u_x}{u_y} = \frac{4}{\tan\theta}$$

$$\boxed{H = \frac{R\tan\theta}{4}}$$

Speed at any Point



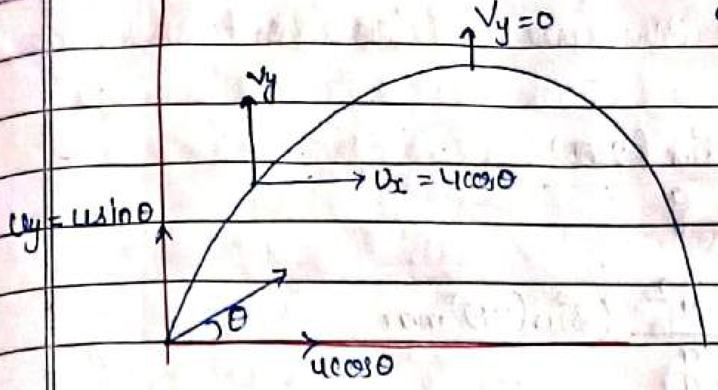


$$\tan\alpha = \frac{u\sin\theta - gt}{u\cos\theta}$$

$\vec{V}$ = Speed = $\sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$

α = Angle of velocity after time 't'

at half of max^m Height



$$\boxed{V_x = u\cos\theta - 1}$$

$$V_y = \frac{u\sin\theta}{\sqrt{2}}$$

$$\boxed{V_x = V_x\hat{i} + V_y\hat{j}}$$

In Vertical dir^n apply 3rd eq^n of mot^n

$$V_y^2 - u_y^2 = 2as$$

$$V_y^2 - (u\sin\theta)^2 = -2g\frac{H_{max}}{2}$$

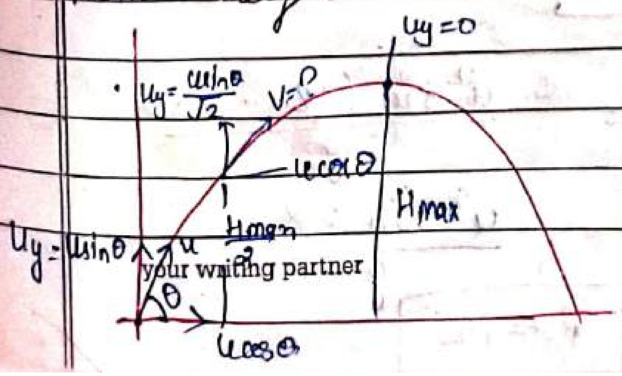
$$V_y^2 = u^2\sin^2\theta - \cancel{2}g\frac{u^2\sin^2\theta}{\cancel{2}(2g)}$$

$$(u\sin\theta)^2 - \frac{(u\sin\theta)^2}{2}$$

$$V_y^2 = \frac{(u\sin\theta)^2}{2}$$

$$\boxed{V_y = \frac{u\sin\theta}{\sqrt{2}}}$$

Que Ball is projected with Speed u at angle θ then final speed of ball at half of max height



$$V = \sqrt{(u\cos\theta)^2 + \left(\frac{u\sin\theta}{\sqrt{2}}\right)^2}$$

$$u\sqrt{\cos^2\theta + \frac{\sin^2\theta}{2}}$$

PAGE

Condition of Maximum Horizontal Range

$$R = \frac{u^2 \sin(2\theta)}{g}$$

$$R_{max} = \frac{u^2}{g}(\sin(2\theta)_{max}$$

$$\boxed{R_{max} = \frac{u^2}{g}}$$

$\sin(2\theta) = 1$

$2\theta = 90$

$\boxed{\theta = 45}$

Que Find Hmax when Range is max^m

R will max^m at $\theta = 45°$

$$H_{max} = \frac{u^2 \sin^2\theta}{2g}$$

$$\frac{u^2}{2g}\left(\frac{1}{\sqrt{2}}\right)^2 = \left(\frac{u^2}{4g}\right)$$

Complementry angle :→

Que Two object is projected with speed u at two diff angle α and β then prove that Range will be same if ($\alpha + \beta = 90°$)

$$\boxed{R_1 = \frac{u^2 \sin 2\alpha}{g}}$$

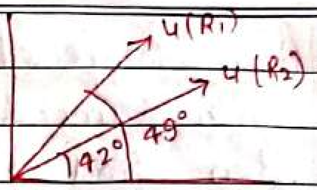
$$R_2 = \frac{u^2 \sin 2\beta}{g} = \frac{u^2 \sin(180 - 2\alpha)}{g}$$

$$\frac{u^2 \sin 2\alpha}{g}$$

$\boxed{R_1 = R_2}$

DATE

α	β
$42°$	$48°$
$30°$	$60°$
$(45+\alpha)$	$(45-\alpha)$
20	70
25	65
$1°$	$89°$

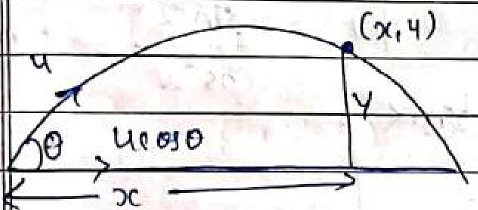


(i) $R_1 = R_2$

(ii) $R_1 > R_2$

(iii) $R_1 < R_2$

Equation of trajectory in projectile motion



Motⁿ along x-axis → $\boxed{x = u\cos\theta\, t}$

Motⁿ along y-axis $\boxed{y = u_y t + \frac{1}{2} a_x t^2}$

$$\boxed{y = u\sin\theta\, t - \frac{1}{2} g t^2} \quad (ii)$$

Putting value of 't' from (i) to eqⁿ (ii)

$$\boxed{t = \frac{x}{u\cos\theta}}$$

$$y = u\sin\theta\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2} g \frac{x^2}{u^2\cos^2\theta}$$

$$\boxed{y = \tan\theta\, x - \frac{g x^2}{2u^2\cos^2\theta}}$$

$$y = x\tan\theta\left[1 - \frac{x}{\frac{2u^2\cos^2\theta\, \sin\theta}{g\cos\theta}}\right]$$

$$\boxed{y = x\tan\theta\left[1 - \frac{x}{R}\right]}$$

$$\boxed{y = ax - bx^2}$$

Parabolic

DATE

Que Ball is projected at two diff angle α and β with same speed such that $\alpha + \beta = 90^\circ$ then find ratio of max^m height and time of flight

$$\alpha + \beta = 90^\circ$$
$$(\because R_1 = R_2)$$

$$\frac{H_1}{H_2} = \frac{\frac{u^2 \sin^2\alpha}{\cancel{2g}}}{\frac{\cancel{u^2} \sin^2\beta}{\cancel{2g}}} = \frac{\sin^2\alpha}{\sin^2\beta} = \frac{\sin^2\alpha}{\sin^2(90-\alpha)} = \frac{\sin^2\alpha}{\cos^2\alpha} = \tan^2\alpha$$

$$\boxed{\frac{T_1}{T_2} = \frac{\frac{\cancel{2u}\sin\alpha}{\cancel{g}}}{\frac{\cancel{2u}\sin\beta}{\cancel{g}}} = \tan\alpha}$$

$\cancel{\tan\alpha}$

$$H_2 H_1 = \frac{u^2\sin^2\alpha}{2g} \times \frac{u^2\sin^2\beta}{2g}$$

$$H_1 H_2 = \frac{(u^2)^2 \sin^2\alpha \sin^2\beta}{4g^2} \frac{(2)^2}{(2)^2}$$

$$H_1 H_2 = \frac{(u^2)^2 (2)^2 \sin^2\alpha \cos^2\alpha}{16 g^2}$$

$$H_1 H_2 = \frac{R^2}{16}$$

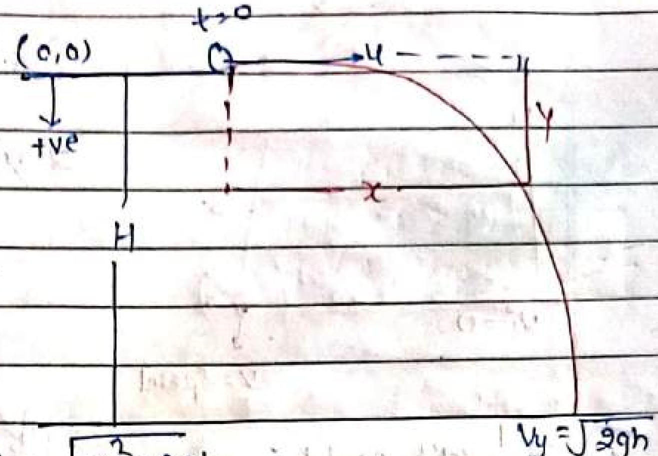
$$\boxed{R = 4\sqrt{H_1 H_2}}$$

β

your writing partner

Horizontal Projectile Motion from Some height

Time of flight :→



Consider motion in y

$S = ut + \frac{1}{2}at^2$

$H = \frac{1}{2}gT^2$

$\boxed{T = \sqrt{\frac{2H}{g}}}$

$|\vec{V}| = \sqrt{u^2 + 2gh}$

$\boxed{\begin{aligned} &V_{strike} = \sqrt{u^2 + 2gh} \\ &V_{at\ time} = \sqrt{u^2 + (gt)^2} \end{aligned}}$

Speed at ground

$\vec{V} = u\hat{i} + \sqrt{2gH}\,\hat{j}$

x axis	y - axis
$u_x = u$	$u_y = 0$
$a_x = 0$	$a_y = +g$ (down)
$v_x = u$	$v_y = gt$ — ①
Uniform Motion	$\boxed{y = \frac{1}{2}gt^2}$
$x = ut$ — ①	

Speed at any time 't'

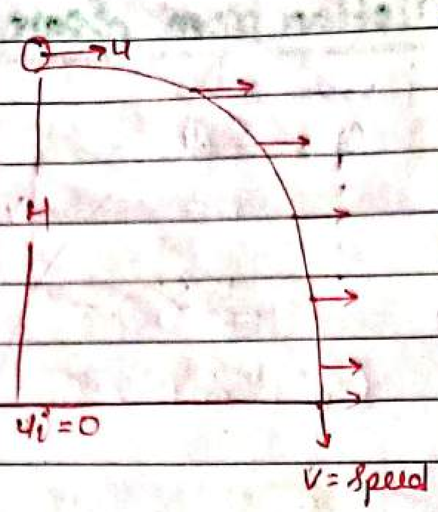
$\vec{V} = u\hat{i} + gt\,\hat{j}$

$|\vec{V}| = \sqrt{u^2 + (gt)^2}$

eqn of Trajectory

$y = \frac{1}{2}g\left(\frac{x}{u}\right)^2 = \frac{1}{2}g\frac{x^2}{u^2}$

Parabolic



Conservation of mechanical energy

$$(K.E + P.E)_i = (K.E + U)_f$$

$$2\cancel{m}gh + \frac{1}{\cancel{2}}\cancel{m}u^2 = 0 + \frac{1}{\cancel{2}}\cancel{m}v_f^2$$

$$v_f^2 = 2gH + u^2$$

$$v_f = \sqrt{u^2 + 2gh}$$

$$\vec{v} = u\hat{i} + \sqrt{2gh}\,\hat{j}$$

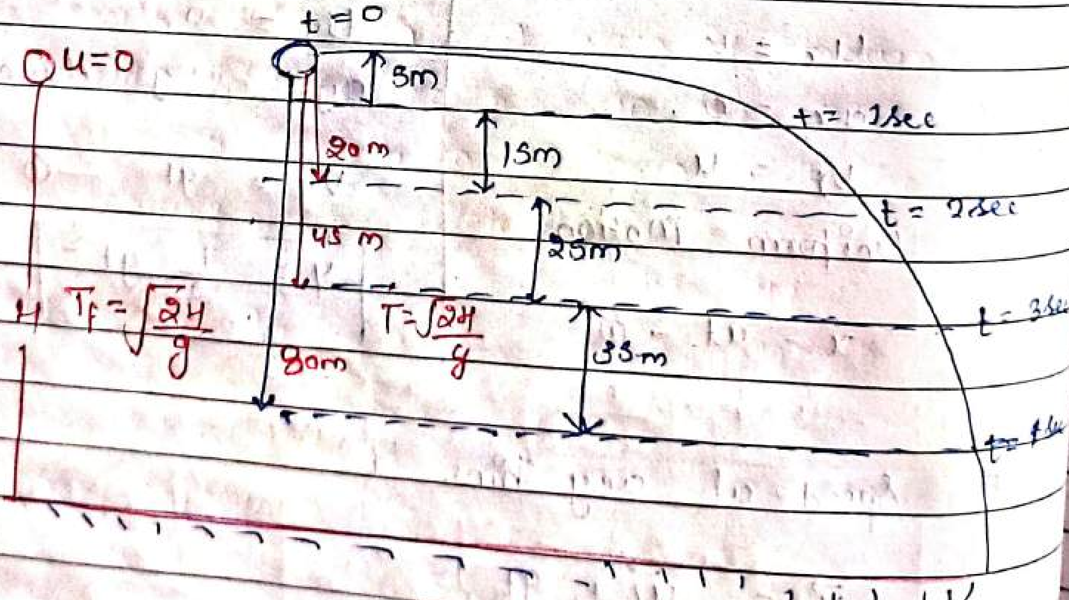
$$|\vec{v}| = \sqrt{u^2 + 2gH}$$

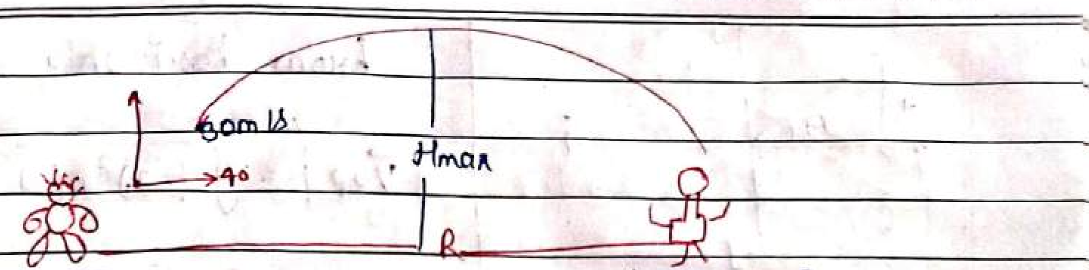
3rd eqn moth in y

$$v^2 - u^2 = 2as$$

$$v_y^2 = 2gh$$

$$\boxed{v_y = \sqrt{2gh}}$$





Ball → $V_B = 40$ m/s

Ball → Path of Ball w.r.t → $V_B = 40$ m/s

$$T_f = \frac{2u_y}{g} = \frac{2\times 30}{10} = 6 \text{ sec}$$

$$H_{max} = \frac{u_y^2}{2g} = \frac{900}{2\times 10} = 45 \text{ m}$$

$$\text{Range} = u_x T_f = 40\times 6 = 240 \text{ m}$$

Relative Motion in plane :->

$$\boxed{V_{AB} = \vec{V_A} - \vec{V_B}}$$

$$\boxed{\vec{V}_{BA} = \vec{V_B} - \vec{V_A}}$$

$$\boxed{\vec{V}_{AB} = -\vec{V}_{BA}}$$

$\vec{V}_{AB} = \vec{u_A} - \vec{u_B}$

$= \vec{V_A} + (-\vec{V_B})$

Que Velocity of Ramlal $\vec{V_R} = -3\hat{i} + 4\hat{j}$ and Velocity of Pinky $\vec{V_P} = 4\hat{i} + 3\hat{j}$ then find Velocity of Ramlal with Respect to Pinky.

$\vec{V_R} = -3\hat{i} + 4\hat{j}$

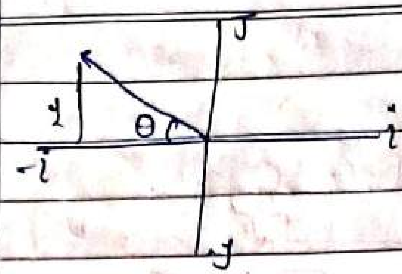
$\vec{V_P} = 4\hat{i} + 3\hat{j}$

$\vec{V}_{RP} = \vec{V_R} - \vec{V_P}$

$-3\hat{i} + 4\hat{j} - (4\hat{i} + 3\hat{j})$

$-3\hat{i} + 4\hat{j} - 4\hat{i} - 3\hat{j}$

$\vec{V}_{RP} = -7\hat{i} + \hat{j}$



$\tan\theta = \frac{1}{7}$

$\theta = \tan^{-1}\left(\frac{1}{7}\right)$

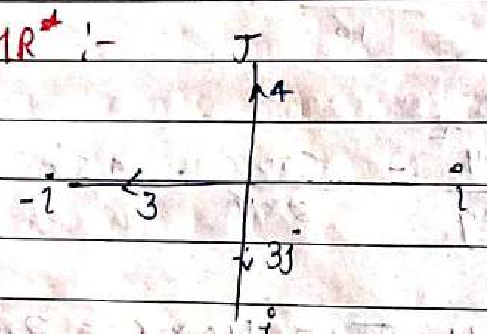
from horizontal

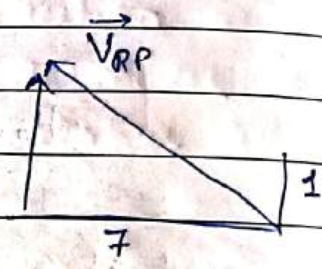
$|\vec{V}_{RP}| = \sqrt{(-7)^2 + (1)^2}$

$= \sqrt{50}$

$\boxed{5\sqrt{2}\ m/s}$ R

MR* :-





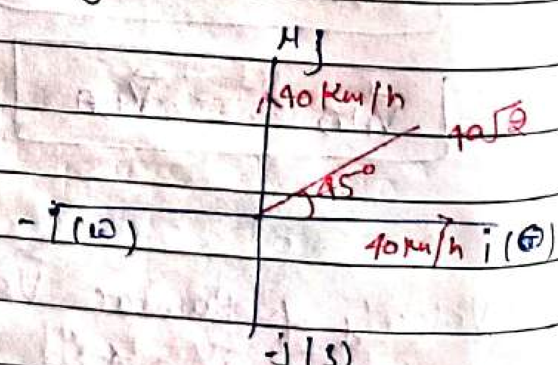
$\boxed{\vec{V}_{RP} = -7\hat{i} + \hat{j}}$ R

Que. A bird is flying with a speed of 40 Km/hr in the north direction. A train is moving with a speed of 40 Km/h in the west direction. A passenger sitting in the train will see the bird moving with velocity ?

$\vec{V}_{BT} = \vec{V}_B - \vec{V}_T$

$40\hat{j} - (-40\hat{j})$

$\boxed{40\hat{j} + 40\hat{i}}$



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$V_R (mR) = 4\,m/s\ \hat{i}$	$\vec{V}_{BR} = \vec{V}_B - \vec{V}_R$	$\vec{V}_{BP} = (4\hat{i}+3\hat{j}) - 3\hat{j}$
$V_P (mR) = 3\,m/s\ \hat{j}$	$\vec{V}_{BR} = 4\hat{i}+3\hat{j}-4\hat{i}$	$= 4\hat{i}$
$V_B (mR) = 4\hat{i}+3\hat{j}$	$3\hat{j}$	
	$V_{PR} = V_P - V_R$	$V_{RP} = \vec{V}_R - \vec{V}_P = 4\hat{i}-3\hat{j}$
	$= 3\hat{j} - 4\hat{i}$	

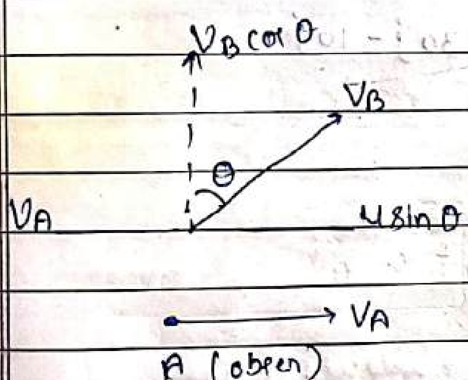
$\vec{V}_{RB} = -\vec{V}_{BR}$

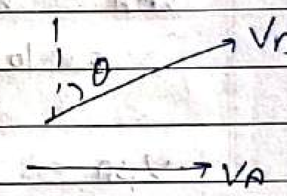
$V_{RB} = -3\hat{j}$

$V_{PB} = -\vec{V}_A$

$= -4\hat{i}$

Que Find relation b/w V_A and V_B so that B appear to move in north w.r.t A.



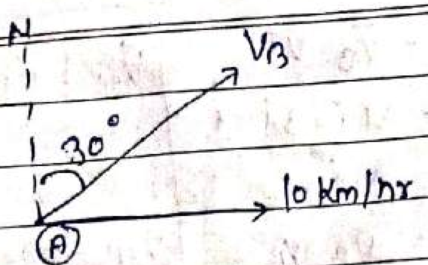


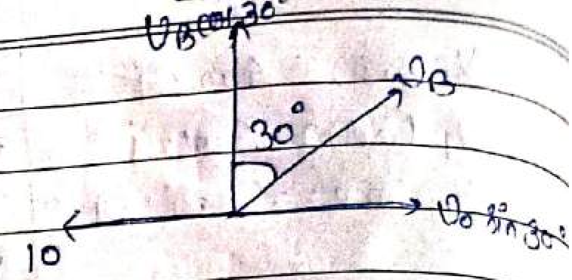
$$\boxed{V_A = V_B \sin\theta}$$

then B appr to move in north

$$\boxed{\frac{V_A}{V_B} = \sin\theta}$$

Que A ship is travelling due east at 10 km/h. Ship heading 30° east of north is always due to north from the first ship. The speed of the second ship in km/h is





$$10 = v_B \sin 30^\circ$$

$$\boxed{v_B = 20}$$

Que A man 'A' moves in the north direction with a speed 10 m/s and another man B moves in E-30° N with 10 m/s. Find the relative velocity of B w.r.t A

$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$$

$$= 10\cos 30\,\hat{i} + 10\sin 30\,\hat{j} - 10\hat{j}$$

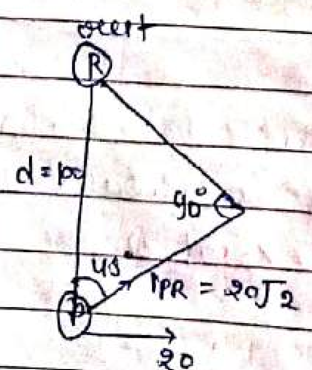
$$V_{BA} = 5\sqrt{3}\,\hat{i} - 5\hat{j}$$

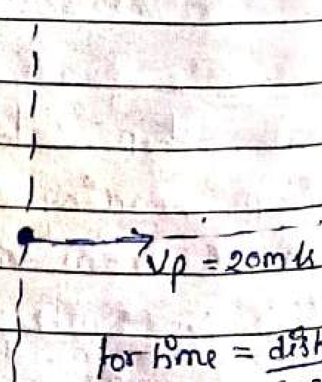
$$|\vec{V}_{BA}| = \sqrt{(5\sqrt{3})^2 + (-5)^2}$$

$$\sqrt{75+25}$$

$$= 10\ m/s\ \text{Ans}$$

Que find minimum sep^n b/w Ramlal and Rinky also find time when they are at min^m sep^n





$$\sin 45^\circ = d_{min}/100$$

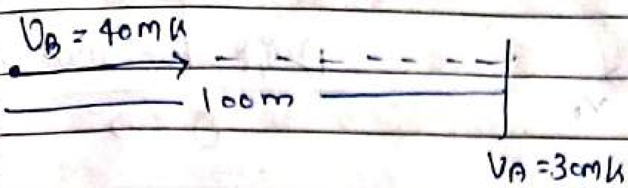
$$d_{min} = \frac{100}{\sqrt{2}} = 50\sqrt{2}\ m$$

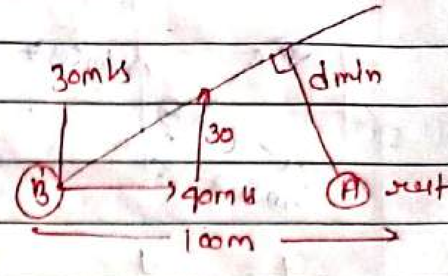
for time $= \frac{\text{dist}}{\text{speed}} = \frac{d}{20\sqrt{2}}$

$$\frac{50\sqrt{2}}{20\sqrt{2}}$$

$$2.5$$

Q. Find min^m sep^n b/w them :→





$\tan\theta = \frac{30}{40} = \frac{3}{4}$

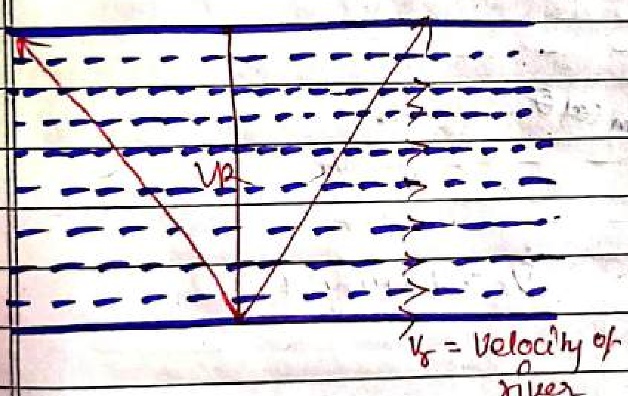
$\boxed{\theta = 37^\circ}$

$\sin 37^\circ = \frac{d_{min}}{100}$

$d_{min} = 100 \times \frac{3}{5}$

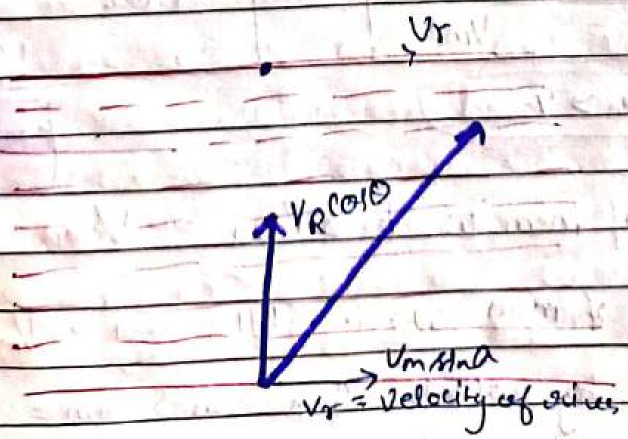
60m

River man Problem :→



There is no effect of velocity of river across the river.

River man problem for minimum time



$time = \frac{d}{V_R\cos\theta}$

$(t_{min}) = \frac{d}{V_{Rm}\,|\cos\theta|_m} = \frac{d}{V_{Rm}}$

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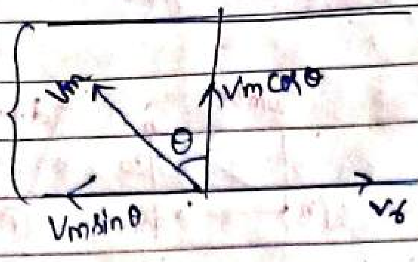


$$\boxed{\text{Drift} = (V_r - V_m \sin\theta)\, t_{min}}$$

Drift for mini time

$\theta = 0^\circ$

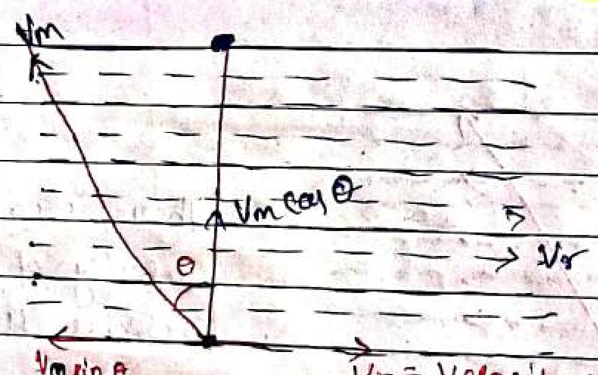
$$t = \frac{d}{V_m \cos\theta}$$

$$t_{min} = \frac{d}{V_m |\cos\theta|_{max}} = \frac{d}{V_m}$$

$|\cos\theta|_{max} = 1$

$\boxed{\theta = 0^\circ}$

River man Problem :- for minimum Path or Zero drift.



V_r = Velocity of river.

If $V_r = V_m \sin\theta$, then drift will be zero.

$$\boxed{\sin\theta = \frac{V_r}{V_m}}$$

$$\text{time} = \frac{d}{V_m \cos\theta}$$

To be possible happen $\boxed{V_m > V_r}$

$\cos\theta = \frac{V_{r}}{V_m}$

Velocity of man w.r.t water = V_{mR}

Velocity of man w.r.t still water = $V_{m\,still\,water}$

Velocity by which man can swim = V_m

Velocity of man by which he can swim = V_m

DATE

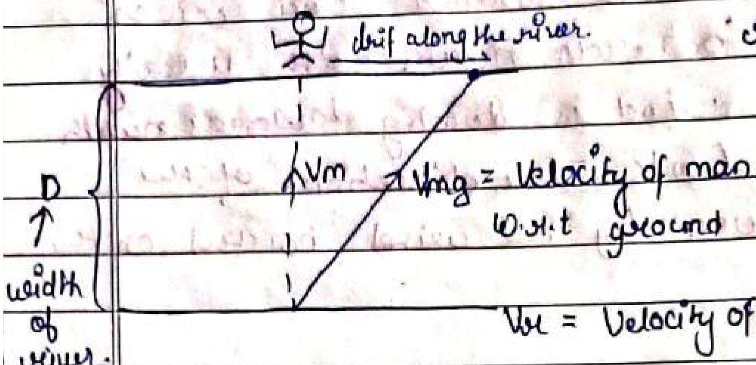
Q. A river is flowing from east to west at a speed of 5m/min. A man on south bank of river capable of swimming 10 m/min in still water wants to swim across the river in shortest time, he should swim

Q. A boat is sailing at a velocity $(3\hat{i}+4\hat{j})$ with respect to ground and water in river is flowing with a velocity $(-3\hat{i}-4\hat{j})$. Relative velocity of the boat with respect to water is

Q. A boat takes 2 hours to go 8 km and come back in still water lake. with water velocity of 4 km/hr. the time taken for going upstream of 8 km and coming back is.

DATE

River man Problem :→ नदी अपने दम पर Cross की जाती है Velocity of river ना ही Support और ना ही oppose करता है



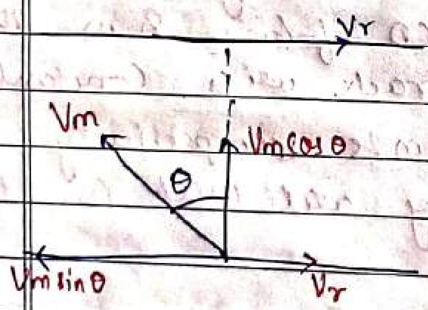
"Minimum time"

$$\boxed{t_{min} = \frac{D}{V_{man}}}$$

Drift in min time – dist along = $V_r t_{min}$ flow of river

$$\boxed{= \frac{V_r D}{V_m}}$$

For minimum path :→



$$\boxed{\text{time to Cross} = \frac{D}{V_m \cos\theta}}$$

If $V_m \sin\theta = V_r$ then (Drift = 0)

$$\boxed{\sin\theta = \frac{V_r}{V_m}}$$

$$\text{Drift} = \frac{D}{V_m \sin\theta}$$

$$(\text{Drift}) = (V_r + V_m \cos\theta)\frac{D}{V_m \sin\theta}$$

V_{man} = Velocity of man by which he Can Swim =
V_{mR} = Velocity of man w.r.t river (river observer)
$V_{m\,water}$ = Velocity of man w.r.t Still water.

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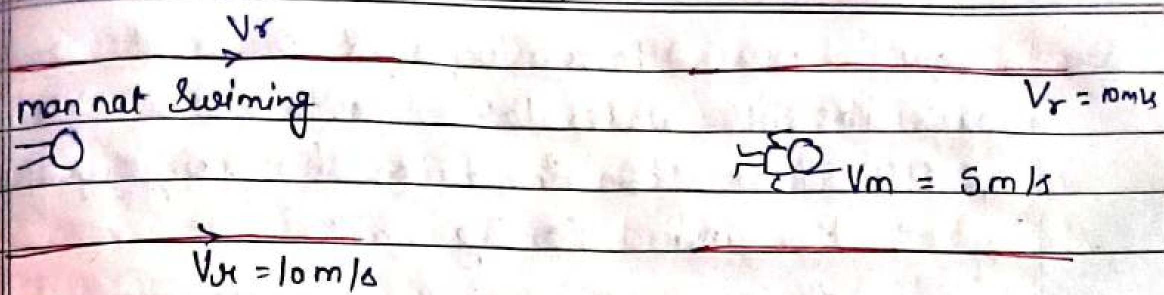
PAGE

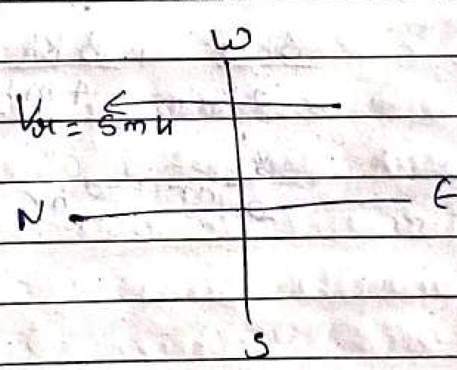
DATE



* Velocity of man w.r.t river $= 0$
* Velocity of man w.r.t ground $= 10$
* Velocity of man w.r.t ground $= 15 m/s$
* Velocity of man w.r.t river $= V_{mg} - V_{river}$
  $15 - 10 = 5$

Qus A river is flowing from east to west at a speed of 5 m/min. A man on south bank of river, capable of swimming 10 m/min in still water wants to swim across the river in shortest time, he should swim

Ans due north.



Qus A boat is sailing at a velocity $(3\hat{i} + 4\hat{j})$ with respect to ground and water in river is flowing with a velocity $(-3\hat{i} - 4\hat{j})$. Relative velocity of the boat with respect to water is

$V_{B\,ground} = 3\hat{i} + 4\hat{j}$

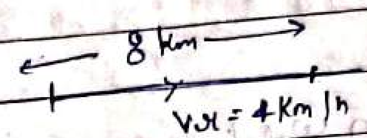
$V_{river} = -3\hat{i} - 4\hat{j}$

$V_{BW} = \vec{V}_{Bg} - \vec{V}_{Wg}$

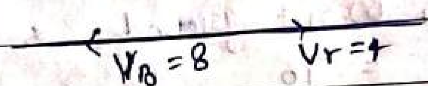
$3\hat{i} + 4\hat{j} - (-3\hat{i} - 4\hat{j})$

DATE

Que A boat takes 2 hour to go 8 km and come back in still water lake. with water velocity of 4 Km/hr. The time taken for going upstream of 8 Km and coming back is



$$\boxed{V_B = \frac{16\ Km}{2\ hr} = 8\ Km}$$



$V_{B_1} = 12\ Km/hr$

$V_{B_2} = 8 - 4$

$= 4\ Km/h$

$t_1 = t_1 + t_2$

$\frac{8\ Km}{12\ Km/hr} + \frac{8\ Km}{4\ Km/hr}$

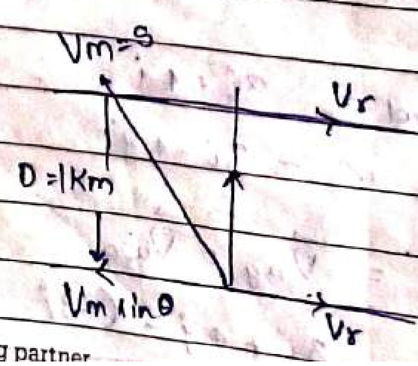
$\frac{8}{12} hr + 2\ hr$

$$\boxed{V_B = \frac{16\ Km}{2\ hr} = 8\ Km/hr}$$

$t = \frac{4}{6} \times 60\ min + 120\ min$

$\boxed{= 160\ min}$ B

Que A boat which has a speed of 5 Km/h in still water crosses a river of width 1 Km along the shortest possible path in 15 minutes. The velocity of the river water in Km/h is



$$t = \frac{D}{V_m \cos\theta}$$

$$\frac{1}{4}\text{ hr} = \frac{1}{5\cos\theta}$$

$$\boxed{\cos\theta = \frac{4}{5}}$$

$$\theta = 37^\circ$$

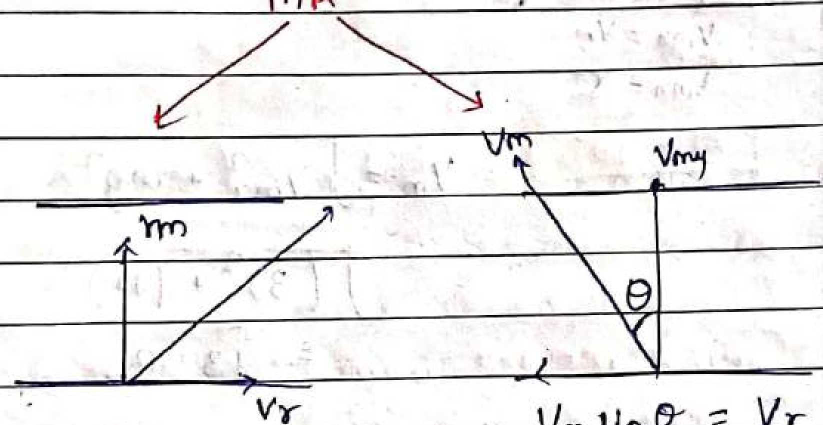
$$V_m \sin\theta = V_r$$

$$5 \sin 37^\circ = V_r$$

$$5 \times \frac{3}{5} = V_r$$

$$\boxed{V_r = 3}\ a$$

MR*



$$t_{min} = \frac{D}{V_m}$$

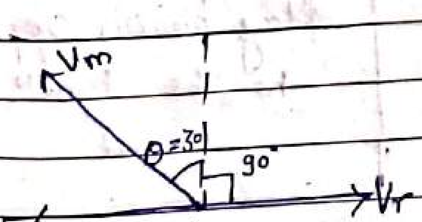
$$\text{Drift} = V_r \frac{D}{V_m}$$

$$V_m \sin\theta = V_r$$

$$\boxed{\text{Drift} = 0}$$

$$\text{time} = \frac{D}{V_m \cos\theta}$$

Que A man wishes to swim across a river 0.5 km wide. If he can swim at the rate of 2 km/hr in still water and the river flows at the rate of 1 km/h. The angle b/w w.r.t the flow of the river along which he should swim so as to reach a point exactly opposite his starting point. Should h



$V_m \sin\theta = V_r$

$$2 \sin\theta = 1$$

$$\sin\theta = \frac{1}{2}$$

$$\boxed{\theta = 30}$$

PAGE